MATH SKILLS

Half-Life

If 100.0 g of carbon-14 decays until only 25.0 g of carbon is left after 11 460 y, what is the half-life of carbon-14?

1. List the given and unknown values.
   
   \[ \text{Given: } \begin{align*}
   &\text{initial mass of sample} = 100.0 \text{ g} \\
   &\text{final mass of sample} = 25.0 \text{ g} \\
   &\text{total time of decay} = 11 460 \text{ y}
   \end{align*} \]

   \[ \text{Unknown: } \begin{align*}
   &\text{number of half-lives} = ? \text{ half-lives} \\
   &\text{half-life} = ? \text{ y}
   \end{align*} \]

2. Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for half-life.

   \[ \frac{\text{total time of decay}}{\text{half-life}} = \text{number of half-lives} \times \frac{\text{number of years}}{\text{half-life}} \]

   \[ \frac{\text{number of years}}{\text{half-life}} = \frac{\text{total time of decay}}{\text{number of half-lives}} \]

3. Calculate how many half-lives have passed during the decay of the 100.0 g sample.

   \[ \text{fraction of sample remaining} = \frac{\text{final mass of sample}}{\text{initial mass of sample}} = \frac{25.0 \text{ g}}{100.0 \text{ g}} = \frac{1}{4} \]

   \[ \text{after one half-life} = \frac{1}{2}; \text{ after two half-lives} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ of sample} \]

   Two half-lives have passed.

4. Solve for the half-life.

   \[ \frac{\text{number of years}}{\text{half-life}} = \frac{11 460 \text{ y}}{2 \text{ half-lives}} = \frac{5730 \text{ y}}{\text{half-life}} \]

   \[ \text{half-life of carbon-14} = 5730 \text{ y} \]

Your Turn to Think

1. What is the half-life of a 100.0 g sample of nitrogen-16 that decays to 12.5 g of nitrogen-16 in 21.6 s?

2. All isotopes of technetium are radioactive, but they have widely varying half-lives. If an 800.0 g sample of technetium-99 decays to 100.0 g of technetium-99 in 639 000 y, what is its half-life?

3. A 208 g sample of sodium-24 decays to 13.0 g of sodium-24 within 60.0 h. What is the half-life of this radioactive isotope?
Half-Life continued

Sample Problem

Thallium-208 has a half-life of 3.053 min. How long will it take for 120.0 g to decay to 7.50 g?

1. List the given and unknown values.

   Given:  
   - half-life = 3.053 min
   - initial mass of sample = 120.0 g
   - final mass of sample = 7.50 g

   Unknown:  
   - number of half-lives = ? half lives
   - total time of decay = ?

2. Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for the total time of decay.

   \[
   \text{total time of decay} = \text{number of half-lives} \times \frac{\text{number of min}}{\text{half-life}}
   \]

3. Calculate how many half-lives have passed during the decay of the 120.0 g sample.

   \[
   \frac{\text{fraction of sample remaining}}{120.0 \text{ g}} = \frac{7.50 \text{ g}}{120.0 \text{ g}} = 0.0625 = \frac{1}{16}
   \]

   After one half-life = \(\frac{1}{2}\); after two half-lives = \(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\);

   After three half-lives = \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}\); after four half-lives = \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}\) of sample. Four half-lives have passed.

4. Solve for the half-life.

   \[
   \text{total time of decay} = 4 \text{ half-lives} \times \frac{3.053 \text{ min}}{\text{half-life}}
   \]

   \[
   \text{total time of decay} = 12.21 \text{ min}
   \]

Your Turn to Think

4. If the half-life of iodine-131 is 8.10 days, how long will it take a 50.00 g sample to decay to 6.25 g?

5. The half-life of hafnium-156 is 0.025 s. How long will it take a 560 g sample to decay to one-fourth its original mass?
MATH SKILLS

**Half-Life continued**

6. Chromium-48 has a short half-life of 21.6 h. How long will it take 360.00 g of chromium-48 to decay to 11.25 g

**Sample Problem**

Gold-198 has a half-life of 2.7 days. How much of a 96 g sample of gold-198 will be left after 8.1 days?

1. List the given and unknown values.
   - **Given:** half-life = 2.7 days
     total time of decay = 8.1 days
     initial mass of sample = 96 g
   - **Unknown:** number of half-lives = ? half-lives
     final mass of sample = ? g

2. Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for the number of half-lives.

   \[
   \text{total time of decay} = \text{number of half-lives} \times \frac{\text{number of days}}{\text{half - life}}
   \]

   \[
   \text{number of half-lives} = \frac{\text{total time of decay}}{\frac{\text{number of days}}{\text{half - life}}}
   \]

3. Calculate how many half-lives have passed during the decay of the 96 g sample.

   \[
   \text{number of half-lives} = \frac{8.1 \text{ days}}{2.7 \text{ days}} = 3.0 \text{ half-lives}
   \]

4. Calculate how much of the sample will remain after 3.0 half-lives.

   \[
   \text{final mass of sample} = \text{initial mass of sample} \times \text{fraction of sample remaining}
   \]

   \[
   \text{fraction of sample remaining after three half-lives} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}
   \]

   \[
   \text{final mass of sample} = 96 \text{ g} \times \frac{1}{8} = 12 \text{ g}
   \]

**Your Turn to Think**

7. Potassium-42 has a half-life of 12.4 hours. How much of an 848 g sample of potassium-42 will be left after 62.0 hours?
**MATH SKILLS**

*Half-Life continued*

8. Carbon-14 has a half-life of 5730 y. How much of a 144 g sample of carbon-14 will remain after $1.719 \times 10^4$ y?

9. If the half-life of uranium-235 is $7.04 \times 10^8$ y and 12.5 g of uranium-235 remain after $2.82 \times 10^9$ y, how much of the radioactive isotope was in the original sample?