Name _____ Date _____ Class _____

Rates of Change

Suppose you have planted a pepper seed in a pot, and you have read that a pepper plant usually does not produce any peppers until it is 0.5 m high. In three weeks, your plant has grown 7.5 cm. Is there any way to predict how long it will probably take for the plant to produce peppers?

To find the answer, consider how much the plant grows in a certain time period. Because the plant has grown 7.5 cm in three weeks, you could write the rate of growth as 7.5 cm/3 weeks, or 2.5 cm/week.

Rates of Change Express Change Over Time

A number like the fraction above, which indicates how quickly a change occurs, is called a rate of change. Rates of change are written as a change over a certain time. The change can be of any type. Speed, for example, describes a change in distance traveled over a certain time. So speed is a rate of change.

Rates of Change are not Necessarily Constant

The 2.5 cm/week growth rate we have assigned to the pepper plant assumes that the plant grows at the same rate from week to week. Often, however, rates of change do not remain constant over time.

For example, consider a bicyclist riding down a hill. At the top of the hill, the bicyclist has a certain rate of change in distance over time. In other words, she has a certain speed. By the time the bicyclist gets to the bottom of the hill, her speed has increased. So her rate of change in distance over time (speed) has changed. This change is called an *acceleration*. Acceleration is another rate of change.

Math Skills

Calculate how long it will take the pepper plant to reach a height of 0.5 m with a constant rate of change.

Solution

1. Determine the rate of change. As discussed above, you can express a rate of change in the form *change/time*.

$$\text{rate of change} = \frac{7.5\text{ cm}}{3.0\text{ weeks}} = \frac{2.5\text{ cm}}{1.0\text{ week}}$$

2. Write a new fraction with the same units as the rate of change, and set it equal to the rate of change found in step one. Because we want to know how long it will take for the pepper plant to reach 0.5 m, we can use x to represent the unknown value, time, and 0.5 m, or 50 cm, for the length.

$$\frac{50\text{ cm}}{x\text{ weeks}} = \frac{2.5\text{ cm}}{1.0\text{ week}}$$

Rates of Change *continued*

3. Solve for *x*. Because you have two fractions that are equal to each other, you can cross-multiply and divide to solve for x.

$$\frac{50\text{ cm} \times 1.0\text{ week}}{2.5\text{ cm}} = \frac{x\text{ weeks} \times 2.5\text{ cm}}{2.5\text{ cm}} \qquad x = 20\text{ weeks}$$

4. Round your answer to the correct number of significant digits. Because 50 cm has one significant digit, you must round your answer to one significant digit. So $x = 20$ weeks (with one significant digit).

Math Skills

A bicyclist riding down a hill goes 1.5 km/h faster each second. How long will it take the bicyclist to increase her speed by 8 km/h?

Solution

1. Determine the rate of change. The rate of change is $\frac{1.5\text{ km / h}}{1\text{ s}}$.

2. Write a new fraction with the same units as the rate of change, and set it equal to the rate of change found in step 1. You want to find how long it will take the bicyclist to speed up by 8 km/h, so you can use *x* for the unknown value, time, and 8 km/h for the speed.

$$\frac{8\text{ km / h}}{x} = \frac{1.5\text{ km / h}}{1\text{ s}}$$

3. Solve for *x*. Cross-multiply and divide to solve for *x*.

$$\frac{8\text{ km / h} \times 1\text{ s}}{1.5\text{ km / h}} = \frac{x\text{ s} \times 1.5\text{km / h}}{1.5\text{km / h}} \qquad x = 5.3333\text{ s}$$

4. Round your answer to the correct number of significant digits. You must round your answer to one significant digit, so $x = 5$ s.

Practice

5. How long would it take the plant in the first example to reach 0.75 m?

6. If the plant grew twice as fast, how long would it take to reach 0.75 m?

7. If the bicyclist from the second example accelerates by 2 km/h every second, how long will it take her to increase her speed by 10 km/h?