**Dimensional Analysis**

Most scientific calculations include numbers paired with units, such as 32 m or 12 km/h. In science, the word *dimension* refers to the physical quality that is measured by a unit of measurement. For example, length is a dimension that can be measured in meters. Other units that measure length include kilometers, miles, and centimeters.

**Rules for Evaluating Equations**

You can use dimensions to evaluate equations by a process called *dimensional analysis*. In dimensional analysis, two rules always apply:

- Quantities must have the same dimensions to be added or subtracted.
- The dimensions on the left side of an equation must be equivalent to the dimensions on the right side of the equation.

You cannot always tell by looking whether the dimensions are equivalent on each side of the equation. In many cases, you have to multiply and cancel to reduce the dimensions to their simplest form.

**Math Skills**

Determine whether the equation \( v = v_0 + at^2 \) is dimensionally correct. The symbols \( v \) and \( v_0 \) are velocities, \( a \) is acceleration, and \( t^2 \) is time squared. Dimensionally, velocity is equivalent to length/time and acceleration is equivalent to length/time².

**Solution**

1. **Reduce the units to their simplest dimensional equivalent.** You can substitute the equivalents given directly into the equation.

   \[
   \frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}} + \frac{\text{length}}{\text{time}^2} = \text{time}^2
   \]

2. **Simplify the dimensional equivalents.** The expression on the right side of the equation can be simplified by canceling \( \text{time}^2 \).

   \[
   \frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}} + \text{length}
   \]
Dimensional Analysis \textit{continued}

3. Check to see if your simplified equation obeys the rules of dimensional analysis. The two dimensional quantities on the right side of the simplified equation, length/time and length, are being added together, even though they are not the same dimension. This goes against the first rule of dimensional analysis. So the original equation is not correct.

Math Skills

Is the equation $5 \text{ m/s} + (10 \text{ m/s}^2 \times 3 \text{ s}) = 1 \text{ mm/s}$ dimensionally correct?

Solution

1. Reduce the units given into their simplest dimensional equivalent. The units m and mm are units of length, and s is a unit of time. These dimensions can be substituted directly into the equation.

\[
\frac{\text{length}}{\text{time}} + \left( \frac{\text{length}}{\text{time}^2} \times \text{time} \right) = \frac{\text{length}}{\text{time}}
\]

2. Simplify the dimensional equivalents. In this case, the longer expression on the left side of the equation can be simplified by cancellation.

\[
\frac{\text{length}}{\text{time}} + \frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}}
\]

3. Check to see if your simplified equation obeys the rules of dimensional analysis. The two dimensions added on the left side of the equation are the same, satisfying the first rule. And both sides of the equation have the same dimension, satisfying the second rule. So, the original equation is dimensionally correct. But this does not mean that the values of the equation are correct.

Practice

1. Is the equation $8 \text{ kg} + 4 \text{ kg} = 12 \text{ kg/s}$ dimensionally correct?
2. Determine whether \( x = \frac{1}{2} at^2 \) is dimensionally correct. The variable \( x \) represents length, \( \frac{1}{2} \) (as with all numbers) does not represent a dimension, \( a \) represents acceleration \( \left( \frac{\text{length}}{\text{time}^2} \right) \), and \( t^2 \) stands for \( \text{time}^2 \).

3. In Einstein’s equation \( E = mc^2 \), \( E \) represents energy, equivalent to \( \left( \frac{\text{mass} \times \text{length}^2}{\text{time}^2} \right) \); \( m \) represents mass; and \( c^2 \) is equivalent to \( \frac{\text{length}^2}{\text{time}^2} \). Is this equation dimensionally correct?