NAME ____________________ DATE __________ CLASS ____________

Problem 10D

HEAT OF PHASE CHANGE

PROBLEM

The world's deepest gold mine, which is located in South Africa, is over 3 km deep. Every day, the mine transfers enough energy by heat to the mine's cooling systems to melt 3.36×10^7 kg of ice at 0.0°C. If the energy output from the mine is increased by 2.0 percent, to what final temperature will the 3.36×10^7 kg of ice-cold water be heated?

SOLUTION

1. DEFINE

Given:

$m_{ice} = m_{water} = m = 3.36 \times 10^7$ kg

$T_i = 0.0°C$

$L_f = 3.33 \times 10^5$ J/kg

$c_{p,w} = 4186$ J/(kg•°C)

$Q' =$ energy added to water $= (2.0 \times 10^{-2})Q$

Unknown: $T_f = ?$

2. PLAN

Choose the equation(s) or situation: First, determine the amount of energy needed to melt 3.36×10^7 kg of ice by using the equation for the heat of fusion.

$$Q = m_{ice}L_f = mL_f$$

The energy added to the now liquid ice (Q') can then be determined.

$$Q' = (2.0 \times 10^{-2})Q = (2.0 \times 10^{-2})mL_f$$

Finally, the energy added to the water equals the product of the water's mass, specific heat capacity, and change in temperature.

$$Q' = mc_{p,w}(T_f - T_i) = (2.0 \times 10^{-2})mL_f$$

Rearrange the equation(s) to isolate the unknown(s):

$$T_f = \frac{(2.0 \times 10^{-2})mL_f}{mc_{p,w}} + T_i = (2.0 \times 10^{-2})\frac{L_f}{c_{p,w}} + T_i$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$T_f = (2.0 \times 10^{-2})\frac{(3.33 \times 10^{-5}\ \text{J/kg})}{(4186\ \text{J/kg•°C})} + 0.0°C = \boxed{1.6°C}$$

4. EVALUATE

Note that the result is independent of the mass of the ice and water. The amount of energy needed to raise the water's temperature by 1°C is a little more than 1 percent of the energy required to melt the ice.

ADDITIONAL PRACTICE

1. Lake Superior contains about 1.20×10^{16} kg of water, whereas Lake Erie contains only 4.8×10^{14} kg of water. Suppose aliens use these two lakes for cooking. They heat Lake Superior to 100.0°C and freeze Lake Erie to

0.0°C. Then they mix the two lakes together to make a "lake shake." What would be the final temperature of the mixture? Assume that the entire energy transfer by heat occurs between the lakes.

2. The lowest temperature measured on the surface of a planetary body in the solar system is that of Triton, the largest of Neptune's moons. The surface temperature on this distant moon can reach a low of –235°C. Suppose an astronaut brings a water bottle containing 0.500 kg of water to Triton. The water's temperature decreases until the water freezes, then the temperature of the ice decreases until it is in thermal equilibrium with Triton at a temperature of –235°C. If the energy transferred by heat from the water to Triton is 471 kJ, what is the value of the water's initial temperature?

3. Suppose that an ice palace built in Minnesota in 1992 is brought into contact with steam with a temperature of 100.0°C. The temperature and mass of the ice palace are 0.0°C and 4.90×10^6 kg, respectively. If all of the steam liquefies by the time all of the ice melts, what is the mass of the steam?

4. In 1992, 1.804×10^6 kg of silver was produced in the United States. What mass of ice must be melted so that this mass of liquid silver can solidify? Assume that both substances are brought into contact at their melting temperatures. The latent heat of fusion for silver is 8.82×10^4 J/kg.

5. The United States Bullion Depository at Fort Knox, Kentucky, contains almost half a million standard mint gold bars, each with a mass of 12.4414 kg. Assuming an initial bar temperature of 5.0°C, each bar will melt if it absorbs 2.50 MJ of energy transferred by heat. If the specific heat capacity of gold is 129 J/kg•°C and the melting point of gold is 1063°C, calculate the heat of fusion of gold.

6. The world's largest piggy bank has a volume of 7.20 m^3. Suppose the bank is filled with copper pennies and that the pennies occupy 80.0 percent of the bank's total volume. The density of copper is 8.92×10^3 kg/m^3.

- **a.** Find the total mass of the coins in the piggy bank.
- **b.** Consider the mass found in (a). If these copper coins are brought to their melting point, how much energy must be added to the coins in order to melt 15 percent of their mass? The latent heat of fusion for copper is 1.34×10^5 J/kg.

7. The total mass of fresh water on Earth is 3.5×10^{19} kg. Suppose all this water has a temperature of 10.0°C. Suppose the entire energy output of the sun is used to bring all of Earth's fresh water to a boiling temperature of 100.0°C, after which the water is completely vaporized.

- **a.** How much energy must be added to the fresh water through heat in order to raise its temperature to the boiling point and vaporize it?
- **b.** If the rate at which energy is transferred from the sun is 4.0×10^{26} J/s, how long will it take for the sun to provide sufficient energy for the heating and vaporization process?