

# Problem 8E

## CONSERVATION OF MECHANICAL ENERGY

### PROBLEM

In 1990, Eddy McDonald of Canada completed 8437 loops with a yo-yo in an hour, setting a world record. Assume that the yo-yo McDonald used had a mass of  $6.00 \times 10^{-2}$  kg. The yo-yo descended from a height of 0.600 m down a vertical string and had a linear speed of 1.80 m/s by the time it reached the bottom of the string. If its final angular speed was 82.6 rad/s, what was the yo-yo's moment of inertia?

### SOLUTION

#### 1. DEFINE

**Given:**  $m = 6.00 \times 10^{-2}$  kg  $h = 0.600$  m  
 $v_f = 1.80$  m/s  $\omega_f = 82.6$  rad/s  
 $g = 9.81$  m/s<sup>2</sup>

**Unknown:**  $I = ?$

#### 2. PLAN

**Choose the equation(s) or situation:** Apply the principle of conservation of mechanical energy.

$$ME_i = ME_f$$

Initially, the system possesses only gravitational potential energy.

$$ME_i = PE_g = mgh$$

When the yo-yo reaches the bottom of the string, this potential energy has been converted to translational and rotational kinetic energy.

$$ME_f = KE_{trans} + KE_{rot} = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Equate the initial and final mechanical energy.

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

**Rearrange the equation(s) to isolate the unknown(s):**

$$\frac{1}{2}I\omega_f^2 = mgh - \frac{1}{2}mv_f^2 \quad I = \frac{2mgh - mv_f^2}{\omega_f^2} = \frac{m(2gh - v_f^2)}{\omega_f^2}$$

#### 3. CALCULATE

**Substitute the values into the equation(s) and solve:**

$$I = \frac{(6.00 \times 10^{-2} \text{ kg})[(2)(9.81 \text{ m/s}^2)(0.600 \text{ m}) - (1.80 \text{ m/s})^2]}{(82.6 \text{ rad/s})^2}$$

$$I = \frac{(6.00 \times 10^{-2} \text{ kg})(8.6 \text{ m}^2/\text{s}^2)}{(82.6 \text{ rad/s})^2} = \boxed{7.6 \times 10^{-5} \text{ kg} \cdot \text{m}^2}$$

#### 4. EVALUATE

Assuming that the yo-yo can be approximated by a solid disk with a central axis of rotation, the yo-yo's moment of inertia is described by the equation  $I = \frac{1}{2}MR^2$ .

From the calculated value for  $I$ , the yo-yo's radius can be found to be 5.0 cm.

$$R = \sqrt{\frac{2I}{M}} = \sqrt{\frac{(2)(7.6 \times 10^{-5} \text{ kg} \cdot \text{m}^2)}{6.00 \times 10^{-2} \text{ kg}}} = 0.050 \text{ m}$$

**ADDITIONAL PRACTICE**

1. In 1993, a group of students in England made a giant yo-yo that was 10 ft in diameter and had a mass of 407 kg. With the use of a crane, the yo-yo was launched from a height of 57.0 m above the ground. Suppose the linear speed of the yo-yo at the end of its descent was 12.4 m/s. If the angular speed at the end of the descent was 28.0 rad/s, what was the yo-yo's moment of inertia?
2. In 1994, a bottle over 3 m in height and with a radius at its base of 0.56 m was made in Australia. Treat the bottle as a thin-walled cylinder rotating about its symmetry axis, which has the same rotational properties as a thin hoop rotating about its symmetry axis. What is the linear speed that the bottle acquires after rolling down a slope with a height of 5.0 m? Do you need to know the mass of the bottle?
3. In 1988, a cheese with a mass of  $1.82 \times 10^4$  kg was made in Wisconsin. Suppose the cheese had a cylindrical shape. The cheese was set to roll along a horizontal road with an undetermined speed. The road then went uphill, and the cheese rolled up the hill until its vertical displacement was 1.2 m, at which point it came to a stop. Assuming that there was no slipping between the rim of the cheese and the ground, calculate the initial linear speed of the cheese.
4. In 1982, a team of ten people rolled a cylindrical barrel with a mass of 64 kg for almost 250 km without stopping. Imagine that at some point during the trip the barrel was stopped at the crest of a steep hill. The barrel was accidentally released and rolled down the hill. If the linear speed of the barrel at the bottom of the slope was 12.0 m/s, how high was the hill? Assume that the barrel's moment of inertia was equal to  $0.80 mr^2$ .
5. A potato with a record-breaking mass of 3.5 kg was grown in 1994. Suppose a child saw this potato and decided to pretend it was a soccer ball. The child kicked the potato so that it rolled without slipping at a speed of 5.4 m/s. The potato rolled up a slope with a  $30.0^\circ$  incline. Assuming that the potato could be approximated as a uniform, solid sphere with a radius of 7.0 cm, what was the distance along the slope that the potato rolled before coming to a stop?
6. In 1992, an artificial egg with a mass of  $4.8 \times 10^3$  kg was made in Australia. Assume that the egg is a solid sphere with a radius of 2.0 m. Calculate the minimum height of a slope that the egg rolls down if it is to reach an angular speed of 5.0 rad/s at the bottom of the slope. What is the translational kinetic energy of the egg at the bottom of the slope?
7. An onion grown in 1994 had a record-breaking mass of 5.55 kg. Assume that this onion can be approximated by a uniform, solid sphere. Suppose the onion rolled down an inclined ramp that had a height of 1.40 m. What was the onion's rotational kinetic energy? Assume that there was no slippage between the ramp and the onion's surface.