Problem 8D

CONSERVATION OF ANGULAR MOMENTUM

PROBLEM

The average distance from Earth to the moon is 3.84×10^5 km. The average orbital speed of the moon when it is at its average distance from Earth is 3.68×10^3 km/h. However, in 1912 the average orbital speed was 3.97×10^3 km/h, and in 1984 it was 3.47×10^3 km/h. Calculate the distances that correspond to the 1912 and 1984 orbital speeds, respectively.

SOLUTION

1. DEFINE **Given:**

$r_{avg} = 3.84 \times 10^5$ km
$v_{avg} = 3.68 \times 10^3$ km/h
$v_1 = 3.97 \times 10^3$ km/h
$v_2 = 3.47 \times 10^3$ km/h

Unknown: $r_1 = ?$ $\quad$ $r_2 = ?$

2. PLAN **Choose the equation(s) or situation:** Because there are no external torques, the angular momentum of the Earth-moon system is conserved.

$$L_{avg} = L_1 = L_2$$
$$I_{avg}\,\omega_{avg} = I_1\,\omega_1 = I_2\,\omega_2$$

If the moon is treated as a point mass revolving around a central axis, its moment of inertia is simply mr^2, and the conservation of momentum expression takes the following form:

$$m_{moon}\,(v_{avg})^2\left(\frac{v_{avg}}{r_{avg}}\right) = m_{moon}\,(r_1)^2\left(\frac{v_1}{r_1}\right) = m_{moon}\,(r_2)^2\left(\frac{v_2}{r_2}\right)$$

Because the mass of the moon is unchanged, the mass term cancels, and the equation reduces to the following:

$$r_{avg}\,v_{avg} = r_1\,v_1 = r_2\,v_2$$

Rearrange the equation(s) to isolate the unknown(s):

$$r_1 = \frac{r_{avg}\,v_{avg}}{v_1} \qquad r_2 = \frac{r_{avg}\,v_{avg}}{v_2}$$

3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$r_1 = \frac{(3.84 \times 10^5\ \text{km})(3.68 \times 10^3\ \text{km/h})}{(3.97 \times 10^3\ \text{km/h})} = \boxed{3.56 \times 10^5\ \text{km}}$$

$$r_2 = \frac{(3.84 \times 10^5\ \text{km})(3.68 \times 10^3\ \text{km/h})}{(3.47 \times 10^3\ \text{km/h})} = \boxed{4.07 \times 10^5\ \text{km}}$$

4. EVALUATE Because angular momentum is conserved in the absence of external torques, the tangential orbital speed of the moon is greater than its average value when the moon is closer to Earth. Similarly, the smaller tangential orbital speed occurs when the moon is farther from Earth.

NAME ______________________ DATE ______________ CLASS ______________

ADDITIONAL PRACTICE

1. Encke's comet revolves around the sun in a period of just over three years (the shortest period of any comet). The closest it approaches the sun is 4.95×10^7 km, at which time its orbital speed is 2.54×10^5 km/h. At what distance from the sun would Encke's comet have a speed equal to 1.81×10^5 km/h?

2. In 1981, Sammy Miller reached a speed of 399 km/h on a rocket-powered ice sled. Suppose the sled, moving at its maximum speed, was hooked to a post with a radius of 0.20 m by a light cord with an unknown initial length. The rocket engine was then turned off, and the sled began to circle the post with negligible resistance as the cord wrapped around the post. If the speed of the sled after 20 turns was 456 km/h, what was the length of the unwound cord?

3. Earth is not a perfect sphere, in part because of its rotation about its axis. A point on the equator is in fact over 21 km farther from Earth's center than is the North pole. Suppose you model Earth as a solid clay sphere with a mass of 25.0 kg and a radius of 15.0 cm. If you begin rotating the sphere with a constant angular speed of 4.70×10^{-3}rad/s (about the same as Earth's), and the sphere continues to rotate without the application of any external torques, what will the change in the sphere's moment of inertia be when the final angular speed equals 4.74×10^{-3} rad/s?

4. In 1971, a model plane built in the Soviet Union by Leonid Lipinsky reached a speed of 395 km/h. The plane was held in a circular path by a control line. Suppose the plane ran out of gas while moving at its maximum speed and Lipinsky pulled the line in to bring the plane home while it continued in a circular path. If the line's initial length is 1.20×10^2 m and Lipinsky shortened the line by 0.80 m every second, what was the plane's speed after 32 s?

5. The longest spacewalk by a team of astronauts lasted more than 8 h. It was performed in 1992 by three crew members from the space shuttle *Endeavour*. Suppose that during the walk two astronauts with equal masses held the opposite ends of a rope that was 10.0 m long. From the point of view of the third astronaut, the other two astronauts rotated about the midpoint of the rope with an angular speed of 1.26 rad/s. If the astronauts shortened the rope equally from both ends, what was their angular speed when the rope was 4.00 m long?

6. In a problem in the previous section, a cherry pie with a radius of 3.00 m and a mass of 17×10^3 kg was rotated on a light platform. Suppose that when the pie reached an angular speed of 3.46 rad/s there was no net torque acting on it. Over time, the filling in the pie began to move outward, changing the pie's moment of inertia. Assume the pie acted like a uniform, rigid, spinning disk with a mass of 16.80×10^3 kg combined with a 0.20×10^3 kg particle. If the smaller mass shifted from a position 2.50 m from the center to one that was 3.00 m from the center, what was the change in the angular speed of the pie?