

# Problem 8B

## ROTATIONAL EQUILIBRIUM

### PROBLEM

In 1960, a polar bear with a mass of  $9.00 \times 10^2$  kg was discovered in Alaska. Suppose this bear crosses a 12.0 m long horizontal bridge that spans a gully. The bridge consists of a wide board that has a uniform mass of  $2.50 \times 10^2$  kg and whose ends are loosely set on either side of the gully. When the bear is two-thirds of the way across the bridge, what is the normal force acting on the board at the end farthest from the bear?

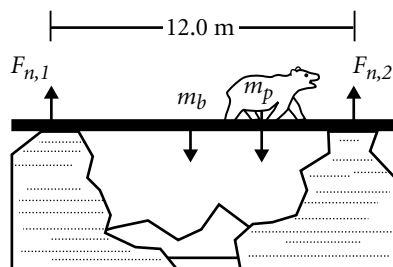
### SOLUTION

#### 1. DEFINE

**Given:**  $m_b =$  mass of bridge  $= 2.50 \times 10^2$  kg  
 $m_p =$  mass of polar bear  $= 9.00 \times 10^2$  kg  
 $\ell =$  length of bridge  $= 12.0$  m  
 $g = 9.81$  m/s<sup>2</sup>

**Unknown:**  $F_{n,1} = ?$

**Diagram:**



#### 2. PLAN

**Choose the equation(s) or situation:**

**Apply the first condition of equilibrium:** The unknowns in this problem are the normal forces exerted upward by the ground on either end of the board. The known quantities are the weights of the bridge and the polar bear. All of the forces are in the vertical ( $y$ ) direction.

$$F_y = F_{n,1} + F_{n,2} - m_b g - m_p g = 0$$

Because there are two unknowns and only one equation, the solution cannot be obtained from the first condition of equilibrium alone.

**Choose a point for calculating net torque:** Choose the end of the bridge farthest from the bear as the pivot point. The torque produced by  $F_{n,1}$  will be zero.

**Apply the second condition of equilibrium:** The torques produced by the bridge's and polar bear's weights are clockwise and therefore negative. The normal force on the end of the bridge opposite the axis of rotation exerts a counterclockwise (positive) torque.

$$\tau_{net} = -(m_b g)d_b - (m_p g)d_p + F_{n,2}d_2 = 0$$

The lever arm for the bridge's weight ( $d_b$ ) is the distance from the bridge's center of mass to the pivot point, or half the bridge's length. The lever arm for the polar bear is two-thirds the bridge's length. The lever arm for the normal force farthest from the pivot equals the entire length of the bridge.

$$d_b = \frac{1}{2}\ell, \quad d_p = \frac{2}{3}\ell, \quad d_2 = \ell$$

The torque equation thus takes the following form:

$$\tau_{net} = -\frac{m_b g \ell}{2} - \frac{2m_p g \ell}{3} + F_{n,2} \ell = 0$$

**Rearrange the equation(s) to isolate the unknowns:**

$$F_{n,2} = \frac{m_b g \ell}{2\ell} + \frac{2m_p g \ell}{3\ell} = \left( \frac{m_b}{2} + \frac{2m_p}{3} \right) g$$

$$F_{n,1} = m_b g + m_p g - F_{n,2}$$

**3. CALCULATE**

**Substitute the values into the equation(s) and solve:**

$$F_{n,2} = \left[ \frac{2.50 \times 10^2 \text{ kg}}{2} + \frac{(2)(9.00 \times 10^2 \text{ kg})}{3} \right] (9.81 \text{ m/s}^2)$$

$$F_{n,2} = (125 \text{ kg} + 6.00 \times 10^2 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F_{n,2} = (725 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F_{n,2} = 7.11 \times 10^3 \text{ N}$$

$$F_{n,1} = (2.50 \times 10^2 \text{ kg})(9.81 \text{ m/s}^2) + (9.00 \times 10^2 \text{ kg})(9.81 \text{ m/s}^2) - 7.11 \times 10^3 \text{ N}$$

$$F_{n,1} = 2.45 \times 10^3 \text{ N} + 8.83 \times 10^3 \text{ N} - 7.11 \times 10^3 \text{ N}$$

$$F_{n,1} = \boxed{4.17 \times 10^3 \text{ N}}$$

**4. EVALUATE**

The sum of the upward normal forces exerted on the ends of the bridge must equal the weight of the polar bear and the bridge. (The individual normal forces change as the polar bear moves across the bridge.)

$$(4.17 \text{ kN} + 7.11 \text{ kN}) = (2.50 \times 10^2 \text{ kg} + 9.00 \times 10^2 \text{ kg})(9.81 \text{ m/s}^2)$$

$$11.28 \text{ kN} = 11.28 \times 10^3 \text{ N}$$

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**ADDITIONAL PRACTICE**

1. The heaviest sea sponge ever collected had a mass of 40.0 kg, but after drying out, its mass decreased to 5.4 kg. Suppose two loads equal to the wet and dry masses of this giant sponge hang from the opposite ends of a horizontal meterstick of negligible mass and that a fulcrum is placed 70.0 cm from the larger of the two masses. How much extra force must be applied to the end of the meterstick with the smaller mass in order to provide equilibrium?
2. A Saguaro cactus with a height of 24 m and an estimated age of 150 years was discovered in 1978 in Arizona. Unfortunately, a storm toppled it in 1986. Suppose the storm produced a torque of  $2.00 \times 10^5 \text{ N}\cdot\text{m}$  that acted on the cactus. If the cactus could withstand a torque of only

$1.2 \times 10^5 \text{ N}\cdot\text{m}$ , what minimum force could have been applied to the cactus keep it standing? At what point and in what direction should this force have been applied? Assume that the cactus itself was very strong and that the roots were just pulled out of the ground.

- 3.** In 1994, John Evans set a record for brick balancing by holding a load of bricks with a mass of 134 kg on his head for 10 s. Another, less extreme, method of balancing this load would be to use a lever. Suppose a board with a length of 7.00 m is placed on a fulcrum and the bricks are set on one end of the board at a distance of 2.00 m from the fulcrum. If a force is applied at a right angle to the other end of the board and the force has a direction that is  $60.0^\circ$  below the horizontal and away from the bricks, how great must this force be to keep the load in equilibrium? Assume the board has negligible mass.
- 4.** In 1994, a vanilla ice lollipop with a mass of  $8.8 \times 10^3 \text{ kg}$  was made in Poland. Suppose this ice lollipop was placed on the end of a lever 15 m in length. A fulcrum was placed 3.0 m from the lollipop so that the lever made an angle of  $20.0^\circ$  with the ground. If the force was applied perpendicular to the lever, what was the smallest magnitude this force could have and still lift the lollipop? Neglect the mass of the lever.
- 5.** The Galápagos fur seals are very small. An average adult male has a mass of 64 kg, and a female has a mass of only 27 kg. Suppose one average adult male seal and one average adult female seal sit on opposite ends of a light board that has a length of 3.0 m. How far from the male seal should the board be pivoted in order for equilibrium to be maintained?
- 6.** Goliath, a giant Galápagos tortoise living in Florida, has a mass of  $3.6 \times 10^2 \text{ kg}$ . Suppose Goliath walks along a heavy board above a swimming pool. The board has a mass of  $6.0 \times 10^2 \text{ kg}$  and a length of 15 m, and it is placed horizontally on the edge of the pool so that only 5.0 m of it extends over the water. How far out along this 5.0 m extension of the board can Goliath walk before he falls into the pool?
- 7.** The largest pumpkin ever grown had a mass of 449 kg. Suppose this pumpkin was placed on a platform that was supported by two bases 5.0 m apart. If the left base exerted a normal force of  $2.70 \times 10^3 \text{ N}$  on the platform, how far must the pumpkin have been from the platform's left edge? The platform had negligible mass.
- 8.** In 1991, a giant stick of Brighton rock (a type of rock candy) was made in England. The candy had a mass of 414 kg and a length of 5.00 m. Imagine that the candy was balanced horizontally on a fulcrum. A child with a mass of 40.0 kg sat on one end of the stick. How far must the fulcrum have been from the child in order to maintain equilibrium?