Problem 8A

TORQUE

PROBLEM

A beam that is hinged near one end can be lowered to stop traffic at a railroad crossing or border checkpoint. Consider a beam with a mass of 12.0 kg that is partially balanced by a 20.0 kg counterweight. The counterweight is located 0.750 m from the beam's fulcrum. A downward force of 1.60×10^2 N applied over the counterweight causes the beam to move upward. If the net torque on the beam is 29.0 N•m when the beam makes an angle of 25.0° with respect to the ground, how long is the beam's longer section? Assume that the portion of the beam between the counterweight and fulcrum has no mass.

SOLUTION

1. DEFINE **Given:**

$m_b = 12.0$ kg

$m_c = 20.0$ kg

$d_c = 0.750$ m

$F_{applied} = 1.60 \times 10^2$ N

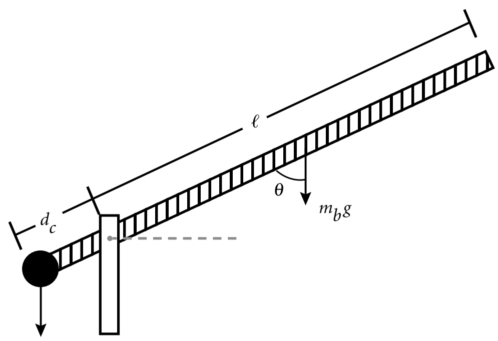
$\tau_{net} = 29.0$ N•m

$\theta = 90.0° - 25.0° = 65.0°$

$g = 9.81$ m/s^2

Unknown: $\ell = ?$

Diagram:



2. PLAN **Choose the equation(s) or situation:** Apply the definition of torque to each force and add up the individual torques.

$$\tau = F d (\sin \theta)$$

$$\tau_{net} = \tau_a + \tau_b + \tau_c$$

where τ_a = counterclockwise torque produced by applied force = $F_{applied}\, d_c (\sin \theta)$

τ_b = clockwise torque produced by weight of beam

$= -m_b\, g \left(\frac{\ell}{2}\right)(\sin \theta)$

τ_c = counterclockwise torque produced by counterweight

$= m_c\, g\, d_c (\sin \theta)$

$$\tau_{net} = F_{applied}\, d_c (\sin \theta) - m_b\, g \left(\frac{\ell}{2}\right)(\sin \theta) + m_c\, g\, d_c (\sin \theta)$$

Note that the clockwise torque is negative, while the counterclockwise torques are positive.

Rearrange the equation(s) to isolate the unknown(s):

$$m_b\, g\left(\frac{\ell}{2}\right) = (F_{applied} + m_c\, g)\, d_c - \left(\frac{\tau_{net}}{\sin\theta}\right)$$

$$\ell = \frac{2\left[(F_{applied} + m_c\, g)d_c - \left(\frac{\tau_{net}}{\sin\theta}\right)\right]}{m_b\, g}$$

3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$\ell = \frac{(2)([1.60 \times 10^2\ \text{N} + (20.0\ \text{kg})(9.81\ \text{m/s}^2)]\ (0.750\ \text{m}) - \left[\frac{29.0\ \text{N}\bullet\text{m}}{\sin 65.0°}\right]}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$

$$\ell = \frac{(2)\ [(1.60 \times 10^2\ \text{N} + 196\ \text{N})(0.750\ \text{m}) - 32.0\ \text{N}\bullet\text{m}]}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$

$$\ell = \frac{(2)\ [356\ \text{N})(0.750\ \text{m}) - 32.0\ \text{N}\bullet\text{m}]}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$

$$\ell = \frac{(2)(2.67 \times 10^2\ \text{N}\bullet\text{m} - 32.0\ \text{N}\bullet\text{m}]}{(12.00\ \text{kg})(9.81\ \text{m/s}^2)}$$

$$\ell = \frac{(2)(235\ \text{N}\bullet\text{m})}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$

$$\ell = \boxed{3.99\ \text{m}}$$

4. EVALUATE For a constant applied force, the net torque is greatest when θ is 90.0° and decreases as the beam rises. Therefore, the beam rises fastest initially.

ADDITIONAL PRACTICE

1. The nests built by the mallee fowl of Australia can have masses as large as 3.00×10^5 kg. Suppose a nest with this mass is being lifted by a crane. The boom of the crane makes an angle of 45.0° with the ground. If the axis of rotation is the lower end of the boom at point *A*, the torque produced by the nest has a magnitude of 3.20×10^7 N•m. Treat the boom's mass as negligible, and calculate the length of the boom.

2. The pterosaur was the most massive flying dinosaur. The average mass for a pterosaur has been estimated from skeletons to have been between 80.0 and 120.0 kg. The wingspan of a pterosaur was greater than 10.0 m. Suppose two pterosaurs with masses of 80.0 kg and 120.0 kg sat on the middle and the far end, respectively, of a light horizontal tree branch. The pterosaurs produced a net counterclockwise torque of 9.4 kN•m about the end of the branch that was attached to the tree. What was the length of the branch?

3. A meterstick of negligible mass is fixed horizontally at its 100.0 cm mark. Imagine this meterstick used as a display for some fruits and vegetables with record-breaking masses. A lemon with a mass of 3.9 kg hangs from the 70.0 cm mark, and a cucumber with a mass of 9.1 kg hangs from the x cm mark. What is the value of x if the net torque acting on the meterstick is 56.0 N•m in the counterclockwise direction?

4. In 1943, there was a gorilla named N'gagi at the San Diego Zoo. Suppose N'gagi were to hang from a bar. If N'gagi produced a torque of -1.3×10^4 N•m about point *A*, what was his weight? Assume the bar has negligible mass.

5. The first—and, in terms of the number of passengers it could carry, the largest—Ferris wheel ever constructed had a diameter of 76 m and held 36 cars, each carrying 60 passengers. Suppose the magnitude of the torque, produced by a ferris wheel car and acting about the center of the wheel, is -1.45×10^6 N•m. What is the car's weight?

6. In 1897, a pair of huge elephant tusks were obtained in Kenya. One tusk had a mass of 102 kg, and the other tusk's mass was 109 kg. Suppose both tusks hang from a light horizontal bar with a length of 3.00 m. The first tusk is placed 0.80 m away from the end of the bar, and the second, more massive tusk is placed 1.80 m away from the end. What is the net torque produced by the tusks if the axis of rotation is at the center of the bar? Neglect the bar's mass.

7. A catapult, a device used to hurl heavy objects such as large stones, consists of a long wooden beam that is mounted so that one end of it pivots freely in a vertical arc. The other end of the beam consists of a large hollowed bowl in which projectiles are placed. Suppose a catapult provides an angular acceleration of 50.0 rad/s^2 to a 5.00×10^2 kg boulder. This can be achieved if the net torque acting on the catapult beam, which is 5.00 m long, is 6.25×10^5 N•m.

a. If the catapult is pulled back so that the beam makes an angle of 10.0° with the horizontal, what is the magnitude of the torque produced by the 5.00×10^2 kg boulder?

b. If the force that accelerates the beam and boulder acts perpendicularly on the beam 4.00 m from the pivot, how large must that force be to produce a net torque of 6.25×10^5 N•m?