

Problem 5C

WORK-KINETIC ENERGY THEOREM

PROBLEM

The Great Pyramid of Khufu in Egypt, used to have a height of 147 m and sides that sloped at an angle of 52.0° with respect to the ground. Stone blocks with masses of 1.37×10^4 kg were used to construct the pyramid. Suppose that a block with this mass at rest on top of the pyramid begins to slide down the side. Calculate the block's kinetic energy at ground level if the coefficient of kinetic friction is 0.45.

SOLUTION

1. DEFINE

Given:

$$m = 1.37 \times 10^4 \text{ kg}$$

$$h = 147 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$\theta = 52.0^\circ$$

$$\mu_k = 0.45$$

$$v_i = 0 \text{ m/s}$$

Unknown: $KE_f = ?$

2. PLAN

Choose the equation(s) or situation: The net work done by the block as it slides down the side of the pyramid can be expressed by using the definition of work in terms of net force. Because the net force is parallel to the displacement, the net work is simply the net force multiplied by the displacement. It can also be expressed in terms of changing kinetic energy by using the work-kinetic energy theorem.

$$W_{net} = F_{net}d$$

$$W_{net} = \Delta KE$$

The net force on the block equals the difference between the component of the force due to free-fall acceleration along the side of the pyramid and the frictional force resisting the downward motion of the block.

$$F_{net} = mg(\sin \theta) - F_k = mg(\sin \theta) - \mu_k mg(\cos \theta)$$

The distance the block travels along the side of the pyramid equals the height of the pyramid divided by the sine of the angle of the side's slope.

$$h = d(\sin \theta)$$

$$d = \frac{h}{\sin \theta}$$

Because the block is initially at rest, its initial kinetic energy is zero, and the change in kinetic energy equals the final kinetic energy.

$$\Delta KE = KE_f - KE_i = KE_f$$

Rearrange the equation(s) to isolate the unknown(s): Combining these equations yields the following expression for the final kinetic energy.

$$KE_f = F_{net}d = mg(\sin \theta - \mu_k \cos \theta) \left(\frac{h}{\sin \theta} \right)$$

$$KE_f = mgh \left(1 - \frac{\mu_k}{\tan \theta} \right)$$

3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$KE_f = (1.37 \times 10^4 \text{ kg})(9.81 \text{ m/s}^2)(147 \text{ m}) \left(1 - \frac{0.45}{\tan 52.0^\circ} \right)$$

$$KE_f = (1.37 \times 10^4 \text{ kg})(9.81 \text{ m/s}^2)(147 \text{ m})(1.00 - 0.35)$$

$$KE_f = (1.37 \times 10^4 \text{ kg})(9.81 \text{ m/s}^2)(147 \text{ m})(0.65)$$

$$KE_f = \boxed{1.3 \times 10^7 \text{ J}}$$

4. EVALUATE Note that the net force, and thus the final kinetic energy, is about two-thirds of what it would be if the side of the pyramid were frictionless.

ADDITIONAL PRACTICE

- The tops of the towers of the Golden Gate Bridge, in San Francisco, are 227 m above the water. Suppose a worker drops a 655 g wrench from the top of a tower. If the average force of air resistance is 2.20 percent of the force of free fall, what will the kinetic energy of the wrench be when it hits the water?
- Bonny Blair of the United States set a world record in speed skating when she skated 5.00×10^2 m with an average speed of 12.92 m/s. Suppose Blair crossed the finish line at this speed and then skated to a stop. If the work done by friction over a certain distance was -2830 J, what would Blair's kinetic energy be, assuming her mass to be 55.0 kg.
- The CN Tower in Toronto, Canada, is 553 m tall, making it the tallest free-standing structure in the world. Suppose a chunk of ice with a mass of 25.0 g falls from the top of the tower. The speed of the ice is 30.0 m/s as it passes the restaurant, which is located 353 m above the ground. What is the magnitude of the average force due to air resistance?
- In 1979, Dr. Hans Liebold of Germany drove a race car 12.6 km with an average speed of 404 km/h. Suppose Liebold applied the brakes to reduce his speed. What was the car's final speed if -3.00 MJ of work was done by the brakes? Assume the combined mass of the car and driver to be 1.00×10^3 kg.
- The summit of Mount Everest is 8848.0 m above sea level, making it the highest summit on Earth. In 1953, Edmund Hillary was the first person to reach the summit. Suppose upon reaching there, Hillary slid a rock with a 45.0 g mass down the side of the mountain. If the rock's speed is

27.0 m/s when it is 8806.0 m above sea level, how much work was done on the rock by air resistance?

- 6.** In 1990, Roger Hickey of California reached a speed 35.0 m/s on his skateboard. Suppose it took 21 kJ of work for Roger to reach this speed from a speed of 25.0 m/s. Calculate Hickey's mass.
- 7.** At the 1984 Winter Olympics, William Johnson of the United States reached a speed of 104.5 km/h in the downhill skiing competition. Suppose Johnson left the slope at that speed and then slid freely along a horizontal surface. If the coefficient of kinetic friction between the skis and the snow was 0.120 and his final speed was half of his initial speed, find the distance William traveled.