

Using Proportions and Cross-Multiplication

Ratios are a powerful tool in science and math. But in order to take full advantage of them, we have to do more than just calculate ratios—we have to put them to work! For example, if you have three bacteria specimens for every student in your class, you know that you will have a ratio of 3 to 1, $\frac{3}{1}$, or 3:1. But this ratio does not tell you the total number of specimens. To find that, you need to use a proportion.

A proportion is a statement of equality between two ratios. This means that the ratios are equal. It also means that the numerator of one ratio multiplied by the denominator of the other ratio is equal to the product of the other numerator and denominator. An example looks like this:

$$\frac{3}{1} = \frac{12}{4}$$

$$3 \times 4 = 1 \times 12$$

$$12 = 12$$

Notice that you are multiplying across the equal sign in your proportion. This process is called *cross-multiplication*. Cross-multiplication is useful because if you know three of the quantities in a proportion, you can find the fourth.

PROCEDURE: To find an unknown quantity in a proportion, set up the numbers you know in equal ratios. Leave the place for the quantity you do not know empty for now. Then cross-multiply the known numerator of one ratio with the known denominator of the other. Then divide this product by your remaining known quantity. The quotient is your answer.

SAMPLE PROBLEM: Find the missing number in this proportion:

$$\frac{5}{20} = \frac{?}{100}$$

Step 1: Cross-multiply the known numerator of one ratio with the known denominator of the other ratio.

$$\frac{5}{20} = \frac{?}{100} \rightarrow 5 \times 100 = 500$$

Step 2: Divide this product with your remaining known quantity.

$$500 \div 20 = 25$$

The missing number in the proportion is 25; $\frac{5}{20} = \frac{25}{100}$

Try It Yourself!

1. Find the unknown quantities in the following proportions:

a. $\frac{3}{8} = \frac{?}{24}$

b. $\frac{21}{?} = \frac{63}{21}$

c. $\frac{?}{3} = \frac{240}{360}$

Using Proportions and Cross-Multiplication, continued

2. Are the following ratios equal? Show your work and then write Yes or No.

a. $\frac{2}{4} = \frac{10}{30}$

b. $\frac{5}{6} = \frac{15}{2}$

c. $\frac{2}{5} = \frac{14}{35}$

3. A cookie recipe calls for 2 eggs for every 15 cookies. How many eggs will you need to bake 45 cookies? *Hint:* Set up your proportion like the one in the sample problem, leaving the place for the unknown quantity blank.

4. The ratio of turtles to fish in a pond is 2 to 5.

a. If the pond has 20 fish, how many turtles are there?

b. How many fish are there if the pond contains 6 turtles?

Challenge Yourself!

5. In a bird sanctuary, 2 out of every 12 birds are eagles, the ratio of hawks to eagles is 1 to 1, the ratio of woodpeckers to hawks is 1 to 2, and the ratio of hawks to cardinals is 1 to 2.

a. The bird sanctuary also has sparrows. How many birds out of every 12 are sparrows?

b. How many cardinals are there for every 180 birds?

