Chapter 12

MG1 Euclidean

1. Gabriel Lame showed that the number of divisions in the Euclidean algorithm is never greater than five times the number of digits in the smaller number.

2. The Euclidean Algorithm is a better method of finding greatest common divisors because it is faster. For two integers, the smallest of which contains 100 digits, the Euclidean Algorithm would require at most 500 steps.

3. Sample answer: To win Euclid's game, first find the greatest common divisor of the two numbers. Next, divide the larger of the two numbers by the greatest common divisor. If the result is even, ask the computer to go first, otherwise take the first turn.

MG1 TruthTbl

1. This truth table would need 16 rows.

2. Answers may vary, here is one possible solution.

P	Q	(~P		Q)		(P		Q)
T	T	F	F	T	F	T	T	T
T	F	F	F	F	F	T	T	F
F	T	T	T	T	T	F	T	T
F	F	T	F	F	F	F	F	F

3. Answers will vary.

4. Answers may vary but may include the following: The Stoics examined a number of ways in which two propositions can be combined to give a third proposition.

5. Two industry applications are a NASA logic table and electrical circuits. The NASA logic table is used to keep track of microswitches on a space shuttle. Sample answer: The logic table is more complicated than a truth table. In the electrical circuit truth table, all values are given in 0's and 1's. A 2-digit value in one column is connected with a 1-digit value in the other column.

MG1 Inductive

1. The first step of a proof by induction is to show that the first case is true (typically, the case is true for $\boldsymbol{n} = 1$).

2. The correct steps should be "***Assume*** that $\boldsymbol{P}(\boldsymbol{k})$ is true, and then prove that $\boldsymbol{P}(\boldsymbol{k} + 1)$ is true."

3. False: $1^2 + 2^2 + 3^2 + \cdots + \boldsymbol{n}^2 = \left[\dfrac{\boldsymbol{n} * (\boldsymbol{n}+1) * (2\boldsymbol{n}+1)}{6}\right]$

4. Solution is presented on the site.

MG1 Farey

1. F_8: $\frac{0}{1}$ $\frac{1}{8}$ $\frac{1}{7}$ $\frac{1}{6}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{2}{7}$ $\frac{1}{3}$ $\frac{3}{8}$ $\frac{2}{5}$ $\frac{3}{7}$ $\frac{1}{2}$ $\frac{4}{7}$ $\frac{3}{5}$ $\frac{5}{8}$ $\frac{2}{3}$ $\frac{5}{7}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{6}{7}$ $\frac{7}{8}$ $\frac{1}{1}$

2. F_4 is $\frac{0}{1}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{1}$. So we have $\frac{0}{1} + \frac{1}{1} = 1$, $\frac{1}{4} + \frac{3}{4} = 1$, and $\frac{1}{3} + \frac{2}{3} = 1$.

3. F_4 is $\frac{0}{1}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{1}$. So we have $\frac{0+1}{1+3} = \frac{1}{4}$, $\frac{1+1}{4+2} = \frac{2}{6} = \frac{1}{3}$, $\frac{1+2}{3+3} = \frac{3}{6} = \frac{1}{2}$, $\frac{1+3}{2+4} = \frac{4}{6} = \frac{2}{3}$, and $\frac{2+1}{3+1} = \frac{3}{4}$.

4. Sample answer: Rational numbers $\frac{\boldsymbol{p}}{\boldsymbol{q}}$ are represented by circles with radius $\frac{1}{\boldsymbol{q}^2}$ directly over the point $\frac{\boldsymbol{p}}{\boldsymbol{q}}$. If the circles representing $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ and $\frac{\boldsymbol{c}}{\boldsymbol{d}}$ are tangent in one point, the circle represented by $\frac{\boldsymbol{a}+\boldsymbol{c}}{\boldsymbol{b}+\boldsymbol{d}}$ is tangent to them both.

MG1 Binary

1. 110001

2. 1101 + 11 = 10000

3. 1101   11 = 100111

4. Our measurement of time, the 12-hour clock, uses a base 12 number system.