

## Chapter 12 Parent Guide

### A Closer Look at Proof and Logic

**Chapter 12 develops the theory behind** proof and logic. It will enable your child to analyze an argument and determine its truth, thereby improving communication skills. If your child goes on to take advanced mathematics courses, this chapter will help prepare them for a deeper, more logical way of thinking.

Every profession can benefit from clear thinking. Lawyers, laboratory scientists, and anyone who will ever have to ask for a raise are among those who require logical thinking skills.

This chapter will take another look at if-then statements studied in Lessons 2.2 and 2.3, examining specific statements for their logical truth value, or validity.

Lessons 12.1 and 12.2 concern logic and how to determine their validity. Lesson 12.3 analyzes the truth value of families of if-then statements. Lesson 12.4 focuses on indirect proof, a style used often in advanced mathematics courses. Lesson 12.5 develops the concept of computer logic.

Though this chapter is short, the concepts contained in it may not come naturally to your child. Most geometry students will be seeing this content for the first time. Encourage your daughter or son to study and work all assigned problems as thoroughly as possible.

Work the following activity with your child to help him or her develop a sense of the logic in conjunction (AND) and disjunction (OR) statements. You may want to briefly review the  $p$  and  $q$  notation defined on page 769 before proceeding with this activity.

### PROBLEM FOR DISCUSSION (See textbook page 776)

In logic, a statement is a sentence that is either true or false. The sentence “Belinda ordered pepperoni on her pizza” is a statement because it must be either true or false. A compound statement is formed when two or more statements are connected. The sentence “John had a soda, and Belinda had tea” is a compound statement. A compound statement, like a simple statement, is either true or false.

1. Read about conjunctions on page 776 and about disjunctions on page 777. How are they alike one another? How are they different?

Conjunctions and disjunctions are both ways to join two different ideas together.

They are different in several ways. For a conjunction to be true, both statements have to be true. For a disjunction to be true, either statement could be true.

For example, for the conjunction, “*Today is Monday and it is raining*” to be true, it has to be Monday and it has to be raining.

For the disjunction, “*Today is Monday or it is raining,*” to be true, it is either Monday, or it is some other day but raining, or it is Monday and it is raining.

2. Compare the truth table on page 776 for conjunctions with the truth table on page 777 for disjunctions. How are they alike? How are they different?

They are only alike when either both  $p$  and  $q$  are true and when both  $p$  and  $q$  are false.

They are different when either  $p$  or  $q$  is false.

3. Consider the conjunction “Today is Tuesday, and tonight is the first varsity track meet” on page 776. Suppose you learn that the statement is true. What can you conclude about each part of the statement?

Each part of the statement must be true because it is a conjunction and conjunctions are only true when both statements are true.

4. Change the compound statement “Today is Tuesday, and tonight is the first varsity track meet” to a disjunction. Make a truth table for it. What can you conclude if the disjunction turns out to be false?

As a disjunction the statement would be “Today is Tuesday or tonight is the first varsity track meet.”

The statement, “Today is Tuesday,” is  $p$ . The statement, “tonight is the first varsity track meet,” is  $q$ .

The truth table would look like the one on page 777 for disjunctions. If it turns out to be false, then it is neither Tuesday, nor is tonight the first varsity track meet.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child’s classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

## Lesson 12.1

10. By the *modus tollens* argument form:  
Therefore, the team did not win on Saturday.
13. If Sims is a man of good moral character, then he is innocent.  
Sims is not a man of good moral character.  
Therefore, Sims is not innocent.  
Argument form: Denying the Antecedent  
Invalid
30. The conclusion is a true statement without regard to the rest of the argument, based on the definitions of a rectangle and a square.
34.  $r$  might be false if one or more of the premises is false.

## Lesson 12.2

12. A ray has only one endpoint and kangaroos can sing. The conjunction is false because one statement (kangaroos can sing) is false.
16. An orange is a fruit or cows have kittens.  
The disjunction is true because one of the statements (an orange is a fruit) is true.
19. Rain does not make the road slippery.
23.  $\triangle ABC$  has two equal angles or  $\triangle ABC$  is isosceles

## Lesson 12.3

12. Conditional: False, since squaring both sides does not necessarily maintain inequality; it can reverse it.  
Converse: If  $a^2 < b^2$  then  $a < b$ .  
False; if  $b = -4$  and  $a = 3$  then  $a^2 < b^2$  but  $a$  is not less than  $b$ .  
Inverse: If  $a \geq b$  then  $a^2 \geq b^2$   
False; if  $b = -4$  and  $a = 3$  then  $a \geq b$  but  $b^2$  is not less than  $a^2$ .  
Contrapositive: If  $a^2 \geq b^2$  then  $a \geq b$ .  
False; if  $a = -4$  and  $b = 3$  then  $a^2 \geq b^2$  but  $a$  is not greater than or equal to  $b$ .

## Chapter 12

- 18.** Pythagorean Theorem: If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.  
Converse: If the square of the length of the hypotenuse of a triangle is equal to the sum of the squares of the lengths of the legs, then the triangle is a right triangle.  
True, this is proven in Lesson 5.4.  
Inverse: If a triangle is not a right triangle, then the square of the length of the hypotenuse is not equal to the sum of the squares of the lengths of the legs.  
True; if a triangle is not a right triangle, then the square of the length of the hypotenuse is greater than or less than the sum of the squares of the lengths of the legs.  
Contrapositive: If the square of the length of the hypotenuse of a triangle is not equal to the sum of the squares of the lengths of the legs, then the triangle is not a right triangle.  
True; The contrapositive is logically equivalent to the original conditional, which is the Pythagorean Theorem.
- 22.** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

### Lesson 12.4

- 14.** Yes; the proof starts by assuming the negation of the statement to be proved. Then a contradiction results after logical arguments. Lastly, the opposite of the assumption is stated to be true because the assumption must be false.
- 20.**  $\overline{BC}$
- 23.** SAS
- 36.** Given:  $m$  and  $n$  are integers  
and  $m^2$  does not divide  $n^2$  with no remainder.  
Suppose  $m$  does divide  $n$  with no remainder (assume the opposite of what is to be proven).  
Then  $n$  can be factored into  $k \cdot m$  for some integer  $k$ .
- $$n = k \cdot m$$
- $$n^2 = k^2 m^2$$
- $$\frac{n^2}{m^2} = k^2$$
- Thus  $m^2$  divides  $n^2$  with no remainder, since  $k^2$  is an integer.  
This is a contradiction.  
Therefore, for two integers  $m$  and  $n$ , if  $m^2$  does not divide  $n^2$  with no remainder, then  $m$  does not divide  $n$  with no remainder.

## Chapter 12

### Lesson 12.5

13. If  $p = 1$ , the output is 1.

17. 1 0 0 1 1

24. 0 1 1 1 1 0 1

30. Read from left to right, one branch at a time. Combine the results of the branches when they flow together.

1.  $p$  OR  $q$

2. The OR gate takes  $r$  and the input from step 1.

$(p$  OR  $q)$  OR  $r$

