

Chapter 11 Parent Guide 11 Taxicabs, Fractals, and More

Not all in the universe fits the Euclidean geometry studied so far in this course. Nature tends to follow fractal geometry. Taxi-cab drivers are familiar with their own taxicab geometry and the field of Astronomy uses hyperbolic geometry.

Chapter 11 provides brief introductions of these and other geometries. An overview of various geometries will give your child a clearer understanding of the structure of geometry and how a change in one part of the structure opens the possibility of an entirely different geometric system.

Because this is an overview of various geometric concepts, no lesson depends on another within the chapter. Each lesson provides new information, much of which your child will be seeing for the first time. These topics will play on your child's imagination.

Lesson 11.1 is on the golden ratio, used often in ancient architecture. Lesson 11.2 is a geometry based on how a taxicab can travel on a street grid. Lesson 11.3 is graph theory which discusses traceable paths. Lesson 11.4 is about topology which is used with knots and stretchable or twistable materials. Lesson 11.5 introduces spherical geometry and two hyperbolic geometries that have applications with planets and space. Fractal geometry in Lesson 11.6 shows up in plants, animals, and land and water formations. Lesson 11.7 discusses distortions created by an overhead projector.

You may enjoy doing the following activity with your child. It is an activity involving taxicab geometry, a geometry you may already have considered as a driver or passenger in a car or bus.

PROBLEM FOR DISCUSSION (See textbook page 706)

In taxicab geometry, points are located on a special kind of map or coordinate grid. The horizontal and vertical lines of the grid represent streets. Unlike points in a traditional coordinate plane, points in a taxicab grid can be only at intersections of two "streets." Thus, the coordinates are always integers.

In taxicab geometry, the distance between two points, known as the taxidistance, is the smallest number of grid units, called blocks, that a taxi must travel to get from one point to the other. On the map, the taxidistance between the two points is 5. How many different ways can the taxi travel from A to B ?

1. Discuss how to count the blocks to find the distance of 5 from A to B .

To count the blocks from point A to point B , you count each unit square. For instance, as you move from $(1, 1)$ to point $(2, 1)$, that is 1 block or unit. You need to find every route from point A to point B that is 5 blocks or units long.

2. How many routes from A to B pass through point $(3, 1)$? How many routes pass through point $(1, 4)$? How many routes are there from point A to point $(2, 1)$ and to point $(2, 2)$? List the possible points that the taxicab could pass through from A to B .

There is one route that passes through point $(3, 1)$ and it is the one that moves 2 units to the right of point A and then 3 units up to point B .

There is one route that passes through point $(1, 4)$ and it is the one that moves 3 units up from point A and then 2 units to the right to point B .

There are 3 routes that go through the point $(2, 1)$ and point $(2, 2)$.

The possible points that a route could go through are: $(1, 1)$, $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 1)$, $(2, 2)$, $(2, 3)$, $(2, 4)$, $(3, 1)$, $(3, 2)$, $(3, 3)$, and $(3, 4)$.

3. Discuss how you might organize your thinking to count all possible routes from A to B . Then count the number of routes.

To organize an answer to this problem, envision a rectangle that connects points A and B . The vertices of this rectangle are $(1, 1)$, $(1, 4)$, $(3, 1)$, and $(3, 4)$.

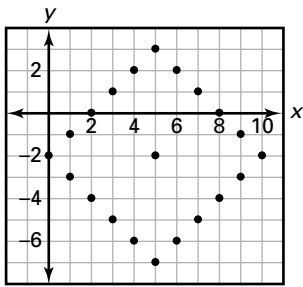
When determining routes, first remember that the taxidistance is 5.

- Then start with the routes that go around the perimeter. There are two of these.
- Then determine how many routes first go 1 right and then 1, 2, or 3 up. There are 3 of these.
- Then determine how many routes first go 1 up and then 1 or 2 right. There are 2 of these.
- Finally, determine how many routes first go up 2 and then right. There are 2 of these.

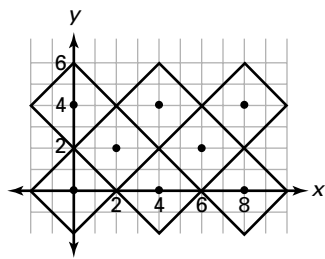
That gives a grand total of 9 different routes with a taxidistance of 5.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

24.



36.



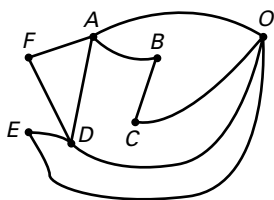
Sample answer:

If one corner of the city grid is $(0, 0)$, then put call boxes at $(4m, 4n)$ and $(2 + 4m, 2 + 4n)$ where m and n are non-negative integers.

Lesson 11.3

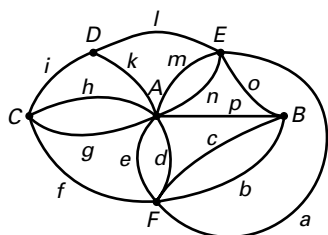
11. Each of the four vertices are odd, so there are no Euler paths or circuits.

16.



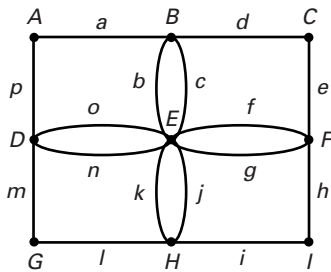
Yes. Each vertex is even, so there is an Euler circuit.

18.



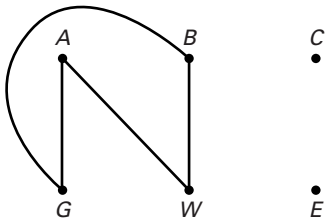
25. 8

34.

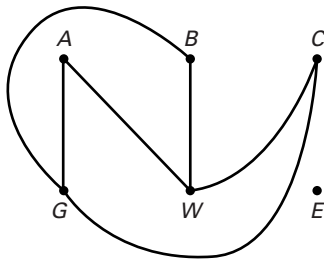


Lesson 11.4

17.–18. Sample answer:

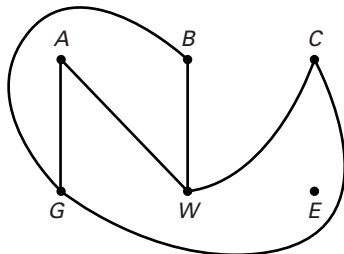


20.



Sample answer:

Note that the curve $BGCW$ is a simple closed curve with A on the inside and E on the outside. By the Jordan Curve Theorem, every curve connecting A to E intersects an already connected utility line. If the edges are redrawn so that E is on the inside of curve $BGCW$, then another simple closed curve, $AGCW$, is formed with E on the inside and B on the outside. Thus, either A or B cannot be connected to E without any intersecting edges.



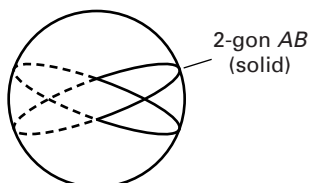
Chapter 11

22. $V = 8, E = 12, F = 6$
 $V - E + F = 8 - 12 + 6 = 2$

26. You must traverse the strip twice in order to get back to the starting point.

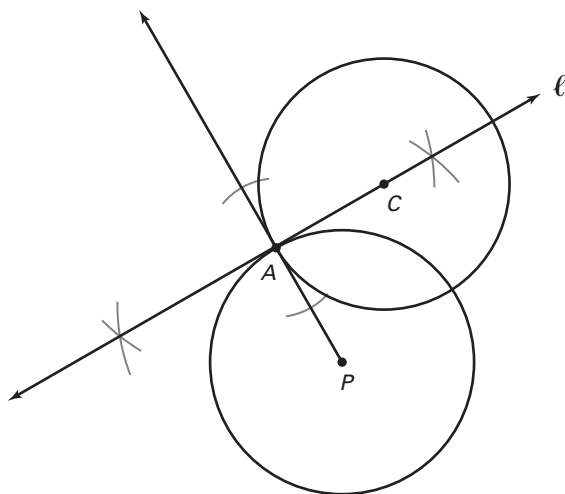
Lesson 11.5

15. Sample answer:



The vertices must be opposite each other on the sphere.

- 19.



29. The sum of the measures of the angles in the triangle is less than 180° . Measure the angles with a protractor to get a fairly accurate reading. In Euclidean geometry the sum of the angle measures in a triangle is 180° , while in spherical geometry it would be greater than 180° .
32. The sum of the measures of the angles in the quadrilateral is less than 360° because there are two triangles in a quadrilateral (divided by a diagonal). In Exercise 29 it was shown that the sum of the angle measures of a triangle is less than 180° , so if you added the sum of the angles of the two triangles, it would be less than 360° .

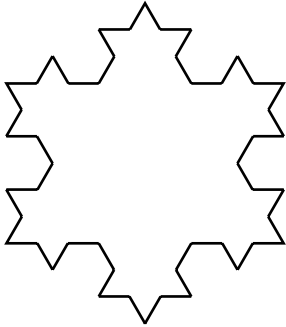
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Lesson 11.6

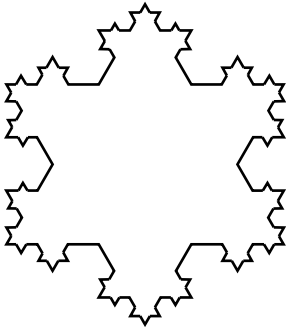
8. $\frac{1}{2}(6)(8) = 24$

14. $9(1.5 + 2 + 2.5) = 9(6) = 54$

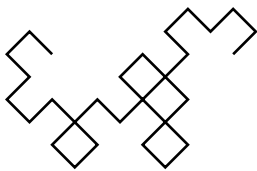
20. Step 2:



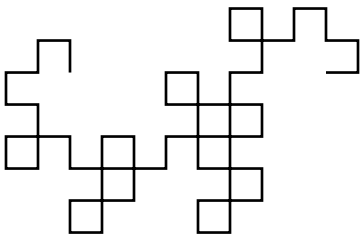
Step 3:



25. Sample answers:



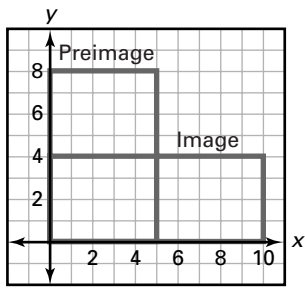
4 iterations



5 iterations

Lesson 11.7

10.



15. a. N

b. $\overrightarrow{NJ}, \overrightarrow{NK}, \overrightarrow{NL}$

18.

