Chapter 10 Parent Guide

Trigonometry

Trigonometry is the study of relations of angles and sides in triangles. The largest angle in a triangle is always opposite the longest side, and the smallest angle is always opposite the shortest side. Moreover, in trigonometry, angle measures can be determined by the ratios of the sides of triangles, and missing side lengths can be determined by using the angle measures.

Many students enjoy learning trigonometry because it is a new and challenging topic.

Chapter 10 reviews the trigonometric definitions, applies two basic trigonometric relations, and also introduces vectors, which are used extensively in physics.

Lessons 10.1 and 10.2 kick off the chapter with a review of the tangent, sine, and cosine ratios. Lesson 10.3 introduces the unit circle, rotations, and graphing the trigonometric functions. Lessons 10.4 and 10.5 employ the law of sines and law of cosines to solve problems involving triangles that are not right triangles. Lesson 10.6 introduces the concept of vectors and the vocabulary connected to them. Lesson 10.7 uses matrices to explore rotations in the coordinate plane.

You might enjoy the following activity with your child. It is an introduction to the sine and cosine ratios.

PROBLEM FOR DISCUSSION (See textbook page 639)

Sine and cosine ratios can be used to solve many types of problems. For example, if you know the length and the approximate angle of the rope, you can estimate the height of a parasailer above the water.

The three most important ratios in trigonometry are the tangent, the sine, and the cosine.

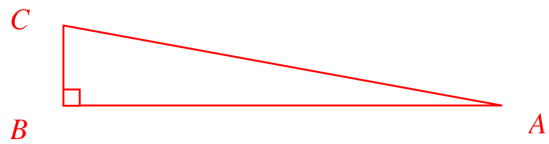
1. Discuss the sine ratio and the cosine ratio. How are they the same? How do they differ?

   The sine and cosine ratio are similar in that they are both ratios of a side of a right triangle and the hypotenuse of a right triangle. The length of the hypotenuse is the denominator in both ratios.

   The difference is that in the sine ratio, the numerator is the length of the side opposite the angle and in the cosine ratio, the numerator is the length of the side adjacent to the angle.

2. How do you know which adjacent side to use in the numerator of the cosine ratio?

   You use the side adjacent, or connected, to the angle you are finding the cosine of. For example, in $\triangle ABC$ below you need to find $\cos A$. The side next to $\angle A$ is AB. It is *not* AC because AC is the hypotenuse. The length of AB would go in the numerator.



3. Study the blue triangles at the top of page 640. What is different about each triangle? What stays the same among the triangles?

The measures of the non-right angles appear to change, as well as the shape of the triangles.

The names of the sides do not change, nor does the measure of the right angle.

4. What happens to the sine as the value of θ increases? Discuss why the sine can never exceed 1.

The sine increases as θ increases until $\theta = 90^{\circ}$. The sine is then equal to 1.

If the sine is bigger than 1, then this means that the ratio of the opposite side to the hypotenuse is bigger than 1. This implies that the opposite side is longer than the hypotenuse. This is impossible because the hypotenuse is always the longest side of a right triangle.

5. What happens to the cosine as the value of θ increases? Discuss why the cosine can never exceed 1.

The cosine decreases as the value of θ increases. The value of the cosine when $\theta = 0$ is 1.

If the cosine is bigger than 1, then this means that the ratio of the adjacent side to the hypotenuse is bigger than 1. This implies that the adjacent side is longer than the hypotenuse. This is impossible because the hypotenuse is always the longest side of a right triangle.

6. What have you learned about sine and cosine from this activity?

Sine and cosine relationships have many similar properties, but are actually inverse relationships.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Chapter 10 - Solutions to Selected Exercises from the Student Textbook

Lesson 10.1

14. $\frac{10}{15} = \frac{2}{3} \approx 0.6667$

21. $\tan 53° \approx 1.33$

25. $\tan^{-1}\frac{7}{5} \approx 54°$

32. $\tan 51° = \frac{46}{x} \Rightarrow (x)(\tan 51°) = 46 \Rightarrow x = \frac{46}{\tan 51°} \approx 37.25$

37. $\tan x = \frac{25}{100} \Rightarrow x = \tan^{-1}\left(\frac{25}{100}\right) \approx 14.0°$

Lesson 10.2

12. $\frac{5}{13} \approx 0.3846$

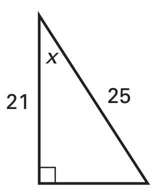
20. Y

33. $83°$

42. $\sin 30° = \cos 60°$ $\quad$ $\sin 65° = \cos 25°$
$\sin 50° = \cos 40°$ $\quad$ $\sin 45° = \cos 45°$

45. $\frac{\sin\theta}{\tan\theta} = \frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}} = \cancel{\sin\theta} \cdot \frac{\cos\theta}{\cancel{\sin\theta}} = \cos\theta$

50. Let x be the angle the slide forms with the tower.



Lesson 10.3

18. -0.4226

25. $\sin 30° = \frac{1}{2} = 0.5$

$\cos 30° = \frac{\sqrt{3}}{2} \approx 0.8660$

35. $x = \cos 300° = 0.5$
$y = \sin 300° \approx -0.8660$

42. $\theta \approx \sin^{-1}(0.9900) \approx 82°$
Another value is $180° - 82° = 98°$.

46. $\cos\theta = -0.7500 \Rightarrow \theta \approx \cos^{-1}(-0.7500) \approx 139°$.
Since $\cos\theta$ is negative only in Quadrants II and III, the other value of θ occurs at an angle of $180° - 139° = 41°$ from horizontal in Quadrant III. So the value of θ is $180° + 41° = 221°$.

Lesson 10.4

10. $m\angle C = 180° - 29° - 27.46° = 123.54°$

$\frac{\sin 29°}{3.28} = \frac{\sin 123.54°}{c} = \Rightarrow (c)(\sin 29°)$

$= 3.28 \sin 123.54° \Rightarrow c = \frac{3.28 \sin 123.54°}{\sin 29°} \approx 5.64$ cm

18. $m\angle Q = 180° - 72° - 36° = 72°$

$$\frac{\sin P}{p} = \frac{\sin Q}{q}$$

$$\frac{\sin 72°}{p} = \frac{\sin 72°}{12}$$

$$12 \cdot \sin 72° = p \cdot \sin 72°$$

$$p = \frac{12 \cdot \sin 72°}{\sin 72°} = 12$$

$$\frac{\sin R}{r} = \frac{\sin P}{p}$$

$$\frac{\sin 36°}{r} = \frac{\sin 72°}{12}$$

$$12 \cdot \sin 36° = r \cdot \sin 72°$$

$$r = \frac{12 \cdot \sin 36°}{\sin 72°} \approx 7.42$$

23. Sample answer: If $a < b \sin A$, then side a is too short to form any triangle, so no triangle is possible.

31. Since $m\angle A < 90°$ and $b \sin A < a < b$ ($b \sin A = 6 \sin 30° = 3$), there are 2 triangles possible.

52. Let A be the angle opposite Oak Street and angle B be the angle opposite 3rd Ave.

$\frac{\sin A}{42} = \frac{\sin 74°}{48.8} \Rightarrow \sin A = \frac{42 \sin 74°}{48.8}$

$\Rightarrow m\angle A = \sin^{-1}\left(\frac{42 \sin 74°}{48.8}\right) \Rightarrow$

$m\angle A \approx 55.82°$

$m\angle B = 180° - 74° - 55.82° = 50.18°$

Lesson 10.5

10. $a^2 = 68.2^2 + 23.6^2 - 2(68.2)(23.6) \cos 87° \Rightarrow a^2 \approx 5039.728 \Rightarrow a \approx 70.99$ units

16. $k^2 = 2^2 + 2^2 - 2(2)(2) \cos 130°$

$k^2 \approx 13.14$

$k \approx 3.63$

$\frac{\sin 130°}{3.63} = \frac{\sin J}{2}$

$\sin J = \frac{2 \cdot \sin 130°}{3.63}$

$m\angle J = \sin^{-1}\left(\frac{2 \cdot \sin 130°}{3.63}\right) \approx 24.96°$

$m\angle L \approx 180° - 130° - 24.96° = 25.04°$

21. law of cosines

27. $b \cdot \cos\theta = \frac{bc_1}{b} \Rightarrow b \cos\theta = c_1$

34. Mark walks $2.8 \times 3 = 8.4$ miles and Stephen walks $4.2 \times 3 = 12.6$ miles.

Let $a = 8.4$, $b = 12.6$ and $m\angle C = 72°$.

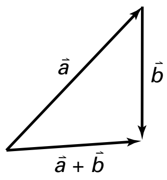
Then $c^2 = (8.4)^2 + (12.6)^2 - 2(8.4)(12.6) \cos 72°$

$c^2 = 163.91$

$c \approx 12.80$ miles

Lesson 10.6

11.



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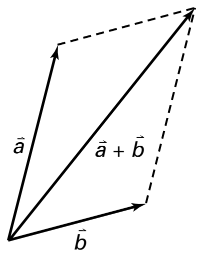
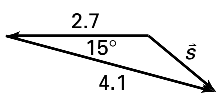


20. $c^2 = (3.75)^2 + (7.5)^2 - 2(3.75)(7.5)\cos 120°$
$c^2 \approx 98.44$
$c \approx 9.92 \Rightarrow |\vec{c}| \approx 9.92$
$\frac{\sin 120°}{9.92} = \frac{\sin M}{3.75} \Rightarrow \sin M = \frac{3.75 \sin 120°}{9.92} \Rightarrow m\angle M = \sin^{-1}\left(\frac{3.75 \sin 120°}{9.92}\right) \Rightarrow m\angle M \approx 19.11°$

25.



$s^2 = (2.7)^2 + (4.1)^2 - 2(2.7)(4.1)\cos 15° \Rightarrow$
$s^2 \approx 2.714 \Rightarrow s \approx 1.65$ mph

Lesson 10.7

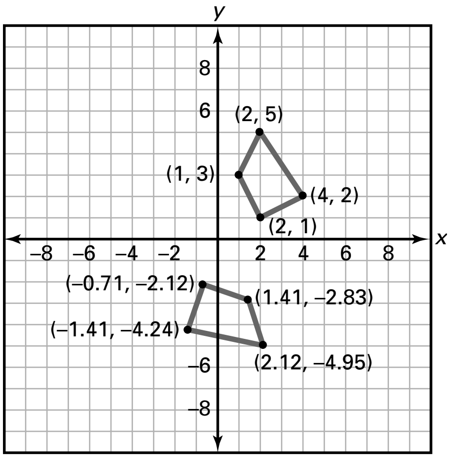
13. $(-3\cos 30° - (-2)\sin 30°, -3\sin 30° + (-2)\cos 30°) \approx (-1.60, -3.23)$

21. 45°

26. $\begin{bmatrix} \cos 320° & -\sin 320° \\ \sin 320° & \cos 320° \end{bmatrix} = \begin{bmatrix} 0.77 & 0.64 \\ -0.64 & 0.77 \end{bmatrix}$

29. $\begin{bmatrix} \cos 225° & -\sin 225° \\ \sin 225° & \cos 225° \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 & 4 \\ 5 & 3 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2.12 & 1.41 & -0.71 & -1.41 \\ -4.95 & -2.83 & -2.12 & -4.24 \end{bmatrix}$



33. $[R_0] = \begin{bmatrix} \cos 0° & -\sin 0° \\ \sin 0° & \cos 0° \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The product of any rotation matrix and the identity matrix gives the rotation matrix.