Chapter 8 Parent Guide
Similar Shapes

Two figures are similar if they have the same shape, but are not necessarily the same size. Engineers, artists, photographers, cartographers, architects, city planners, and toy manufacturers are among the many professionals who use models geometrically similar to a life-size figure.

Proportion is used to show that two figures are similar. Also, if two figures are similar, then their sides are proportionate. Proportion will be used throughout this chapter.

Recall that the symbol for congruence is an equal symbol with a squiggly line over it. The squiggly line stands for similarity. In other words, congruent figures are both similar and equal in size.

Chapter 8 opens with projecting an image on a screen, moves to similar polygons, then focuses on using similar triangles. The chapter ends with a study of area and volume of similar figures.

Lesson 8.1 involves dilations and their scale factors. Lesson 8.2 defines and uses similar polygons. Lessons 8.3 and 8.4 develop triangle similarity postulates and theorems involving side-splitting segments. Lesson 8.5 shows how to use triangle similarity for indirect measurement, used when a direct measurement is impossible to reach. Finally, Lesson 8.6 involves a study comparing area and volume of similar figures.

The following activity is based on the beginning part of Lesson 8.2. You may want to do this activity with your child to reinforce the meaning of similarity.

PROBLEM FOR DISCUSSION (See textbook page 507)

When a figure undergoes a dilation (Lesson 8.1), the preimage and image have the same shape but are not necessarily the same size. They are said to be similar.

1. How can you tell if two triangles are similar to one another? What corresponding parts must be congruent? What corresponding parts can be different?

   Two triangles are similar if and only if one is congruent to the image of the other by a dilation.

   A dilation is an example of a transformation that is not rigid. Dilations preserve the shape of an object, but they may change its size.

   Corresponding angles of similar triangles must be congruent.

   Corresponding sides of similar triangles need not be congruent, but they must be proportional.

2. Is it possible for two quadrilaterals to have congruent angles and not be similar? Discuss your reasoning.

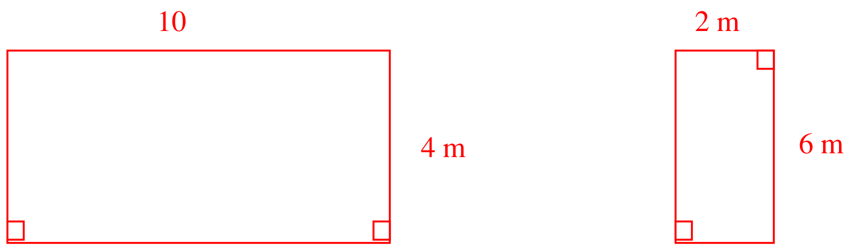
Yes, it is possible for two quadrilaterals to have congruent angles and not be similar. Consider a square and a rectangle. Both have four right angles but their sides are not in proportion. Therefore, they are not similar.

3. Are all squares similar to one another? Discuss your reasoning.

Yes, all squares are similar to one another. All squares have four right angles so their corresponding angles are congruent. Squares have four congruent sides; therefore, all sides of squares are proportional.

4. Are all rectangles similar to one another? Discuss your reasoning.

No, all rectangles are not similar to one another. Although their angles are congruent, it is not necessarily true that their sides will be in proportion. Consider the example:



The ratio between the longer sides is 10:6 or 5:3. The ratio between the shorter sides is 4:2 or 2:1. Therefore, these rectangles are not similar.

5. Are all triangles similar to one another? Discuss your reasoning.

No, all triangles are not similar to one another because there are infinitely many triangles with different angle measures. Consider a 45-45-90º triangle and a 60-60-60º triangle. None of their angles are congruent and therefore are not similar.

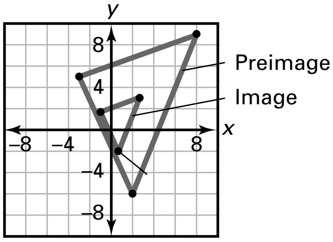
6. Why are all circles similar to one another?

All circles are similar because all circles have the same shape and their radii are in proportion.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

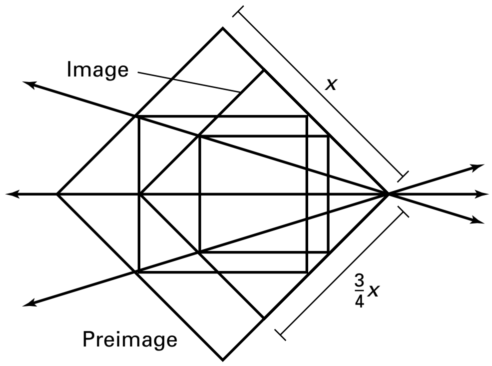
Lesson 8.1

15. $\left(-1, \frac{5}{3}\right), \left(\frac{8}{3}, 3\right), \left(\frac{2}{3}, -2\right)$



20. -1

23.



28. Slope of preimage: $m = \frac{1 - 3}{3 - (-2)} = -\frac{2}{5}$.
Endpoints of image are $(-10, 15)$ and $(15, 5)$.
So, $m = \frac{5 - 15}{15 - (-10)} = \frac{-10}{25} = -\frac{2}{5}$.

37. 4

40. The scale factor is $\frac{12}{4} = 3$. Thus, the coordinates of the image are (0, 3), (3, 3), (6, 6), and (3, 6), respectively.

Lesson 8.2

13. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$

18. $\angle S \cong \angle U$, $\angle Q \cong \angle V$, and $\angle R$, $\angle T$ are right angles, so corresponding angles are congruent. $\frac{30}{19.5} = \frac{16}{10.4} = \frac{34}{22.1}$, so corresponding sides are proportional. Thus $\triangle RSQ \sim \triangle TUV$.

27. $\frac{x}{2} = \frac{1.8}{3} \Rightarrow 3x = 3.6 \Rightarrow x = 1.2$

33. $\frac{2}{5}x = \frac{56}{10} \Rightarrow \frac{2x}{5} = \frac{56}{10} \Rightarrow 20x = 280 \Rightarrow x = 14$

39. False. It is true that $\frac{3}{4} = \frac{6}{8}$. But, $\frac{3+1}{4+1} = \frac{4}{5}$ and $\frac{6+1}{8+1} = \frac{7}{9}$. So, $\frac{3+1}{4+1} \neq \frac{6+1}{8+1}$.

43. Let x be the estimate of the number of fish in the lake. Then, $\frac{8}{100} = \frac{300}{x} \Rightarrow 8x = 30{,}000 \Rightarrow x = 3750$ $\Rightarrow$ 3750 fish is the estimate.

Lesson 8.3

11. $\triangle LJK \sim \triangle CBA$; SSS

19. Yes. AB and AC are proportional to AD and AE, respectively. Also, the included angle, $\angle A$, is shared by both $\triangle ABC$ and $\triangle ADE$, so by the SAS Similarity Theorem, $\triangle ABC \sim \triangle ADE$.

28. Since $\overline{ST} \parallel \overline{VW}$, $\angle UST \cong \angle UVW$ by the Corresponding Angles Postulate. Also, $\angle U$ is shared by both triangles, so by the AA Similarity Postulate, $\triangle UST \sim \triangle UVW$.

36. Yes. Since Biata multiplied each length by 5 to get the sides of her triangle, then by the SSS Similarity Theorem, the two triangles are similar.

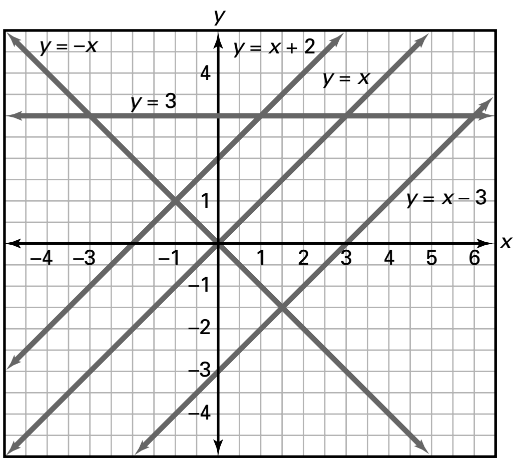
Lesson 8.4

15. $\frac{7}{7+x} = \frac{6}{15} \Rightarrow 6(7+x) = 105 \Rightarrow 42 + 6x = 105$
$\Rightarrow 6x = 63 \Rightarrow x = \frac{21}{2} = 10.5$

20. $\frac{5}{x+1} = \frac{2x+4}{3x-1} \Rightarrow (2x+4)(x+1) = 5(3x-1)$
$\Rightarrow 2x^2 + 6x + 4 = 15x - 5$
$\Rightarrow 2x^2 - 9x + 9 = 0 \Rightarrow (2x-3)(x-3) = 0$
$\Rightarrow x = \frac{3}{2}$ or $x = 3$

25. By the Two-Transversal Proportionality Corollary,
$\frac{7}{2.5} = \frac{6x+3y}{3}$ and
$\frac{2.5}{5} = \frac{3}{5x+2y}$, or $15x + 7.5y = 21$ and
$12.5x + 5y = 15$. Solving this set of equations,
$x = \frac{2}{5}$ and $y = 2$.

32.



The slope of $y = -x$ is -1, the negative reciprocal of the slope of the other lines, which is 1.

38. The length of the string for the note G is $\frac{8}{9} \cdot 40 = 35\frac{5}{9}$ cm. If the distance from the string for the higher F to the string for the G is y, then $\frac{40}{40+y}$ $= \frac{20}{35\frac{5}{9}} \Rightarrow 1422\frac{2}{9} = 800 + 20y \Rightarrow 622\frac{2}{9} = 20y \Rightarrow y = 31\frac{1}{9} \Rightarrow$ The string should be placed $31\frac{1}{9}$ cm from the higher F.

Lesson 8.5

11. $\frac{h}{20} = \frac{10}{12} \Rightarrow 12h = 200 \Rightarrow h = 16\frac{2}{3}$
$\Rightarrow$ Height is $16\frac{2}{3}$ ft

16. $\frac{7.2}{x} = \frac{8}{10} \Rightarrow 8x = 72 \Rightarrow x \Rightarrow 9$

23. $\frac{1}{2}(LK)(JK) = \frac{1}{2}(9)(12) = 54 \Rightarrow$ Area of $\triangle JKL$ is 54.
$\frac{1}{2}(JK)(KM) = \frac{1}{2}(12)(16) = 96$
$\Rightarrow$ Area of $\triangle MKJ$ is 96.

42. $\frac{\text{Height of person}}{\text{Distance from person to mirror}} = \frac{\text{Height of dinosaur}}{\text{Distance from dinosaur to mirror}}$
$\frac{5}{3} = \frac{x}{12} \Rightarrow 3x = 60 \Rightarrow x = 20$
The dinosaur is 20 ft tall.

Lesson 8.6

9. Since the ratio of the corresponding sides is
$\frac{7 + 3}{3} = \frac{10}{3}$, the ratio of the perimeters is $\frac{10}{3}$.

12. The ratio of the areas is $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$, so the area of
VWXYZ is $50 \cdot \frac{25}{16} = 78.215 \text{ m}^2$.

14. $\frac{7}{5}$

21. $\left(\frac{7}{9}\right)^3 = \frac{343}{729}$

32. $\sqrt[3]{\frac{64}{125}} = \frac{4}{5} \Rightarrow \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

45. Let x be the original length and y be the original width. Then $xy = 400$. So,
$$\begin{aligned}(1.75)(x)(1.5)(y) &= (1.75)(1.5)xy \\ &= (1.75)(1.5)(400) \\ &= 1050\end{aligned}$$
$\Rightarrow$ Area of expanded lot is 1050 m^2.