

## Chapter 7 Parent Guide

### Surface Area and Volume

**Surface area is the total area of the faces** of a polyhedron and volume is the total capacity of the polyhedron. Students who have a firm grasp on the concepts in the previous chapter should not experience major difficulties.

Understanding how to calculate surface area and volume can be helpful in many real-world applications. For instance, volume is used to communicate the capacity of aquariums, swimming pools, and containers. Surface area is helpful when estimating the amount of paint needed to paint a house, room, or fence, or to know how much wrapping is needed to cover a box.

Chapter 7 is organized according to types of solids.

Except for Lessons 7.1 and 7.7, each lesson discusses the surface area and volume of prisms, pyramids, cylinders, cones and spheres. Lesson 7.1 explores ratios of surface area to volume and Lesson 7.7 investigates symmetry in three-dimensional figures.

A common mistake is that students sometimes confuse the formulas for surface area and volume. Stress that surface area will add areas and volume multiplies the base by the height. Encourage your child to be aware of what the problem is asking.

You can help your child recognize patterns in the formulas for pyramids and cones by doing the following activity together. This activity will help your son or daughter realize how information in this chapter can be categorized.

### PROBLEM FOR DISCUSSION (See textbook page 460)

As a volcano erupts and deposits lava and ash over a period of time, it forms a cone. Volcanic cones may be different shapes and sizes, depending on factors such as the rate at which the lava and ash are deposited, how fast the lava cools, etc.

How does a cone compare to a pyramid?

1. Discuss the difference between a right cone and an oblique cone.

**In a right cone, the altitude intersects the base of the cone at its center.**

**If it does not intersect at the center, then it is an oblique cone.**

2. Discuss the definition of a cone and its parts noted in the diagram.

**A cone is a three dimensional figure that is similar to a cylinder. The difference is that a cylinder has two circular bases and a cone has only one.**

**The lateral surface of a cone is connected by the vertex and the base and its altitude is the perpendicular from the vertex to the base.**

3. Look at the pyramids on page 445. Which parts of a pyramid have the

same name as the parts of a cone? How is a cone like a pyramid?  
How is it different?

A pyramid has a vertex and a base like a cone.

Cones and pyramids are similar because they both have bases and vertices.

They are different in that a pyramid is a rigid figure and a cone is a rounded figure.

4. The formula for the surface area of a regular pyramid is given on page 446, and the formula for that of a right cone is on page 462. How are the two formulas different? How are they the same?

The surface area for both is the sum of the area of the base and the area of the lateral surface.

The difference is in how you calculate the area of the base. For the area of the base of a right cone, you use the formula  $\pi r^2$  since the base is a circle. For the area of the base of a regular pyramid, you use  $s^2$  since the base is a square.

5. The formula for the volume of a regular pyramid is on page 448, and the formula for that of a right cone is on page 463. How are the two formulas different? How are they the same?

The volume for both is the same;  $\frac{1}{3} Bh$ .

The difference is that for the area of the base of a right cone you use the formula  $\pi r^2$ , since the base of a right cone is a circle. For the area of the base of a regular pyramid, you use  $s^2$  since the base of a regular pyramid is a square.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

## Lesson 7.1

15. 48

$$28. V = s^3 = 1000 \Rightarrow s = 10 \text{ cm}$$

$$S = 6s^2 = 6(10)^2 = 600 \text{ cm}^2$$

$$\frac{S}{V} = \frac{600}{1000} = 0.6$$

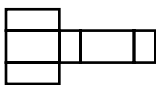
33. Sample answer: Minimize the surface area. Assuming the item(s) to be mailed have a fixed volume, you would want to minimize the amount of packaging to reduce the weight of the package.

36. The prism with dimensions  $n \times n \times n$ , or a cube, has the smallest surface area for a given volume.

40. Sample answer: The thousands of air sacs make the surface area of the lungs large, thereby maximizing the ability of the lungs to absorb oxygen. This surface area is much greater than that of the skin, so the human lungs can absorb enough oxygen to support a much larger body than the skin could absorb.

## Lesson 7.2

10. Sample answer:



$$15. V = (17)(23) = 391 \text{ in}^3$$

$$18. B = (16)(9) = 144 \text{ units}^2$$

$$p = 2(16 + 9) = 50 \text{ units}$$

$$L = hp = (10)(50) = 500 \text{ units}^2$$

$$S = L + 2B = 500 + 2(144) = 788 \text{ units}^2$$

$$V = (16)(9)(10) = 1440 \text{ units}^3$$

23. A side of the hexagon measures 8 cm and the perimeter is  $(6)(8) = 48$  cm.

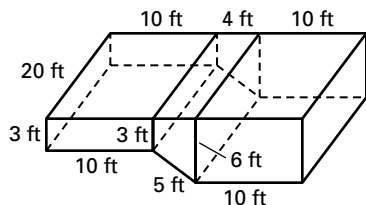
$$S = (20)(48) + 2\left(\frac{1}{2}\right)(4\sqrt{3})(48) = 960 + 192\sqrt{3}$$

$$\approx 1292.55 \text{ cm}^2$$

## Chapter 7

27.  $V = \left(\frac{7(6+8)}{2}\right)18 = 882 \text{ m}^3$

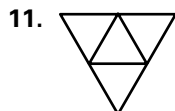
36. The prism can be divided into 2 rectangular prisms and one trapezoidal prism.



$$V = (20)(10)(3) + \frac{1}{2}(6+3)(4)(20) + (20)(10)(6)$$

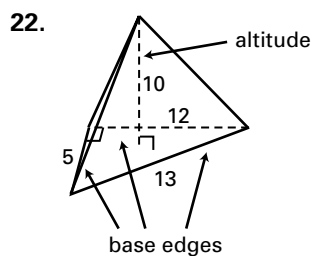
$$= 600 + 360 + 1200 = 2160 \text{ ft}^3 \approx 16,119.4 \text{ gal.}$$

### Lesson 7.3



15.  $S = \frac{1}{2}\ell p + B = \left(\frac{1}{2}\right)(7)(4)(6) + 6^2 = 120 \text{ units}^2$

18.  $V = \frac{1}{3}Bh = \frac{1}{3}(7)(9)(8) = 168 \text{ m}^3$



$$V = \frac{1}{3}Bh = \frac{1}{3} \times \frac{(5 \times 12)}{2} \times 10 = 100 \text{ units}^3$$

28. The slant height  $\ell$  is the hypotenuse of a right triangle with legs of length 2 and 3.

$$\ell = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.61 \text{ units.}$$

$$\text{Area of } \triangle AEB = \frac{1}{2}(3+4)\sqrt{13} \approx 12.62 \text{ units}^2$$

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- 33.** The base of the pyramid is an equilateral triangle which has a perimeter of 12 units, so its side length is 4 units and its area is  $\frac{1}{2} \cdot 4 \cdot 2\sqrt{3} = 4\sqrt{3}$  units<sup>2</sup>. The volume of the pyramid is 8 units<sup>3</sup>, so  $8 = \frac{1}{3}Bh = \frac{1}{3} \cdot 4\sqrt{3} \cdot h$ . Thus,  $h = \frac{3 \cdot 8}{4\sqrt{3}} = 2\sqrt{3} \approx 3.46$  units

### Lesson 7.4

**15.**  $S = 2\pi(4)(15 + 4) = 152\pi \approx 477.5$  units<sup>2</sup>

**24.**  $V = \pi r^2 h = 54\pi$

$$r = \sqrt{\frac{54}{h}} = \sqrt{\frac{54}{6}} = 3 \text{ units}$$

**29.**  $V = \pi r^2 h$

Doubling the height of a cylinder doubles the volume of the cylinder.

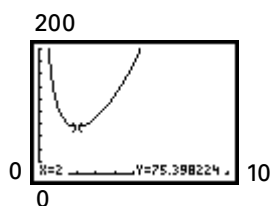
**34.**  $\pi r^2 h = 16\pi$

$$h = \frac{16}{r^2}$$

$$S = 2\pi r h + 2\pi r^2$$

$$= 2\pi r \frac{16}{r^2} + 2\pi r^2$$

$$= \frac{32\pi}{r} + 2\pi r^2$$



$$r = 2 \text{ units}$$

$$S \approx 75.40 \text{ units}^2$$

$$h = 4 \text{ units}$$

### Lesson 7.5

**11.**  $S = \pi(6)(10) + \pi(6)^2 = 96\pi \approx 301.59$  cm<sup>2</sup>

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19.  $\ell = \sqrt{40^2 + 9^2} = 41$   
 $S = \pi r \ell + \pi r^2 = 369\pi + 81\pi = 450\pi \approx 1413.72 \text{ units}^2$

25.  $V = \frac{1}{3}\pi r^2 h$   
 $= \frac{1}{3}\pi(7)^2(45) \approx 2309.07 \text{ units}^3$

37.  $\sqrt{91}$

38.  $\frac{1}{3}\pi \times 3^2 \times \sqrt{91} \approx 89.91$

46. Cylindrical glass:

$$r = \frac{7.2}{2} = 3.6 \text{ cm}$$

$$V = \pi r^2 h = \pi(3.6)^2(14.6) \approx 594 \text{ cm}^3$$

Conical glass:

$$r = \frac{7.5}{2} = 3.75 \text{ cm}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3.75)^2(18.4) \approx 271 \text{ cm}^3$$

### Lesson 7.6

12.  $S = 4\pi(13.41)^2 \approx 719.3124\pi \approx 2259.79 \text{ units}^2$   
 $V = \frac{4}{3}\pi(13.41)^3 \approx 3215.33\pi \approx 10,101.25 \text{ units}^3$

23.  $S = 4\pi(2y)^2 = 16y^2\pi$   
 $V = \frac{4}{3}\pi(2y)^3 = \frac{32}{3}y^3\pi$

28.  $A = \pi r^2$   
 $S = 4\pi r^2 = 4A = 4 \times 32.30 = 129.2 \text{ units}^2$

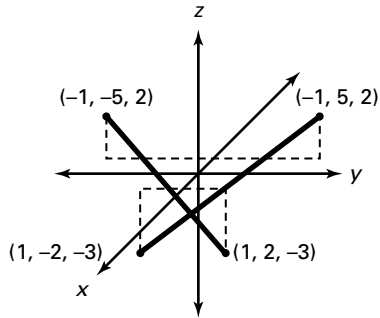
32. The largest ball would have a diameter equal in length to the edge of the cube.

$$V = \frac{4}{3}\pi(6)^3 = 288\pi \approx 904.78 \text{ in}^3$$

37.  $S = 4\pi r^2 = 4\pi\left(\frac{2.9}{2}\right)^2 \approx 26.42 \text{ in}^2$   
 $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{2.9}{2}\right)^3 \approx 12.77 \text{ in}^3$

Lesson 7.7

12.



19.  $(4, -2, -3)$

23. back-right-bottom,  $(-4, 4, -1)$

30. A cylinder with  $h = 10$  and  $r = 5$ , centered on the z-axis.

37.

