Chapter 5 — Parent Guide
Perimeter and Area

Though most of the formulas in this chapter will be familiar**,** your child will need to apply the properties of quadrilaterals and triangles to use them.

Often it is impractical to measure each side of a polygon to find its perimeter. If you know, for example, that a fence forms a rectangle, you need only to know its length and width to find the perimeter or area. However, first you need to know that it is, indeed, a rectangle.

Chapter 5 revisits all the familiar perimeter and area formulas of polygons and circles. It also includes the Pythagorean theorem and distance formulas learned in algebra. Coordinate geometry is revisited and geometric probability is examined in the later lessons.

Lessons 5.1 through 5.3 focus on perimeter and area of polygons and circles. The perimeter of a circle is called the circumference. Lessons 5.4 and 5.5 apply the Pythagorean theorem and its converse to solve problems involving right triangles, including special right triangles. Lesson 5.6 discusses the distance formula and uses it with a special technique to estimate the area of a circle. Calculus, an advanced mathematics course, is based on this special measuring technique. Lesson 5.7 uses the distance formula to locate vertices and midsegments of triangles and quadrilaterals. Finally, Lesson 5.8 focuses on geometric probability, based on area.

Do the following activity with your child to illustrate how geometry is sometimes used in science.

PROBLEM FOR DISCUSSION (See textbook page 353)

The surface of Earth consists of about 30 percent land and 70 percent water. Assuming that a comet or asteroid would be equally likely to strike anywhere on Earth, what is the probability that such an object would strike land instead of water?

1. Discuss the meaning of probability, including the meaning of 0% chance and 100% chance that an event would occur.

Probability is a number from 0 to 1 (or from 0 to 100 percent) that indicates how likely an event is to occur.

A probability of 0 (or 0%) indicates that the event *cannot* occur. For example, if all you have in your sock drawer is white socks and someone asks, “what is the probability that if I go to your sock drawer I will pull out a blue sock,” the probability is 0.

A probability of 1 or (100%) indicates that the event *must* occur. Consider the same sock drawer, if someone asks, “what is the probability that I will pick out a white sock,” the probability is 1.

2. Discuss why a comet or asteroid would be more likely to strike water than land. Discuss the probability of it striking land. Discuss the probability of it striking water.

A comet or asteroid would be more likely to strike water than land because there is more water on the earth than there is land.

There is a 70% chance that the comet or asteroid would hit water because the earth is 70% water.

There is a 30% chance that the comet or asteroid would hit land because the earth is 30% land.

3. To the nearest whole percent, what is the probability that a meteor lands at the edge of your nearest pond or lake? Explain how you determined your answer.

Discuss that for a meteor to land at a local pond or lake near your house is most unlikely considering the size of the world. Therefore, the probability is more than likely less than 1%.

4. Suppose 100 asteroids or comets fall to Earth during the next few years. Estimate how many of them will strike water and how many will strike land.

This is directly related to the amount of land the Earth has and the amount of water the Earth has.

Since 70% of the earth is water, then approximately 70% of the 100 comets, or 70 comets, will hit water.

Since 30% of the earth is land, then approximately 30% of the 100 comets, or 30 comets, will hit water.

5. What geometric principle is used to solve these kinds of problems?

To solve these kinds of problems, you need to use the area of certain objects. For instance, when answering questions about the earth, you deal with the area of the earth that is land and the area of the earth that is water.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 5.1

9. Perimeter $ADLI = 2(15) + 2(11) = 52$ in.

16. Area $BCHG = (BC)(CH) = (8)(4) = 32 \text{ in}^2$

21. The base is the length of the segment between the points (3, 1) and (9, 1), $|9 - 3| = 6$ units. The height is the length of the segment between (3, 1) and (3, 7), $|7 - 1| = 6$ units. The area is: $A = bh = 6 \cdot 6 = 36 \text{ units}^2$.

25. Let h be the height.
Then $b = 2h + 3$ and
$A = (2h + 3)h = 2h^2 + 3h = 27$
$2h^2 + 3h - 27 = 0$
$(h - 3)(2h + 9) = 0$
$h = 3$ or $h = -\frac{9}{2}$
The height cannot be negative, so $h = 3$.
$b = 2(3) + 3 = 9$ cm
$P = 2b + 2h = 18 + 6 = 24 \text{ cm}^2$

34. $(6)(15) = 90 \text{ ft}^2$ of panel are needed. Each panel must cover an area of $90 \div 2 = 45 \text{ ft}^2$ and be $45 \div 10 = 4.5$ ft wide.

36.

base	height	perimeter	area
1	5625	11,252	5625
2	2812.5	5629	5625
5	1125	2260	5625
20	281.25	602.5	5625
30	187.5	435	5625
40	140.625	361.25	5625
50	112.5	325	5625
60	93.75	307.50	5625
70	80.36	300.71	5625
75	75	300	5625
80	70.31	300.63	5625
90	62.5	305	5625

The minimum amount of fencing needed is 300 feet.

Lesson 5.2

14. $A = (2)(11) = 22 \text{ units}^2$

18. $A = \frac{1}{2}(25 + 40)(20) = 650 \text{ units}^2$

24. $A = (JK)(FK) = (10)(7) = 70 \text{ units}^2$

36. $b = |3 - 1| = 2$. $h = |3 - 0| = 3$.
$A = 2 \cdot 3 = 6 \text{ units}^2$

51. Sample answer:
Given: kite $ABCD$ with $\overline{AC} \perp \overline{BD}$,
Prove: Area $ABCD = \frac{1}{2}(BD)(AC)$

Statement	Reasons
$ABCD$ is a kite with $\overline{AC} \perp \overline{BD}$.	Given
Area kite $ABCD$ = Area $\triangle ABD$ + Area $\triangle BCD$	Sum of Areas Postulate
Area kite $ABCD = \frac{1}{2}(AX)(BD) + \frac{1}{2}(XC)(BD)$	Area of a Triangle Formula
Area kite $ABCD = \frac{1}{2}(BD)(AX + XC)$	Distributive Property
Area kite $ABCD = \frac{1}{2}(BD)(AC)$	Segment Addition Postulate

Lesson 5.3

10. $C = 2\pi r = \pi d = \pi(18) \approx (3.14)(18) \approx 56.5$
$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{18}{2}\right)^2 = \pi(9)^2 \approx (3.14)(81)$
≈ 254.3

16. $50\pi = 2\pi r$
$r = 25$

22. $A = \pi \cdot (1.5)^2 = 2.25\pi \approx 7.07$

34. $C = 2(12.5)\pi \approx 78.5 \text{ ft}$

Lesson 5.4

9. $c^2 = 10^2 + 15^2 = 325 \Rightarrow c = \sqrt{325} = 5\sqrt{13}$

16. The altitude divides the triangle into 2 right triangles. Use the Pythagorean Theorem to find the third side of each, then add to find the third side of the larger triangle.

$17^2 = 8^2 + b^2$ $\quad$ $10^2 = 8^2 + e^2$
$289 = 64 + b^2$ $\quad$ $100 = 64 + e^2$
$225 = b^2$ $\quad$ $36 = e^2$
$b = \sqrt{225} = 15$ $\quad$ $e = \sqrt{36} = 6$

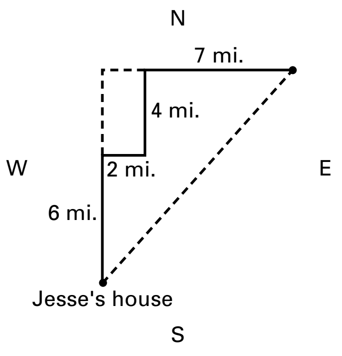
The length of the third side of the outer triangle is: $b + e = 15 + 6 = 21$
The perimeter is: $P = 21 + 17 + 10 = 48$ units.

21. $26^2 > 24^2 + 7^2$; obtuse

26. $c^2 = 5^2 + 5^2 = 50$. Therefore,
diagonal $= \sqrt{50} = 5\sqrt{2}$ cm ≈ 7.07 units

29. Area shaded region $= \frac{1}{2}\pi\left(\frac{9}{2}\right)^2 = \frac{\pi a^2}{8}$
$a^2 + 21^2 = 29^2$
$a^2 = 841 - 441 = 400$
$A = \frac{\pi \cdot 400}{8} = 50\pi \approx 157.1$ units^2

41.



The distance from Jesse's house is the length of the hypotenuse of the right triangle, with sides of length $6 + 4 = 10$ miles and $2 + 7 = 9$ miles.
$c^2 = 10^2 + 9^2$
$c^2 = 181$
$c = \sqrt{181} \approx 13.5$ miles

Lesson 5.5

15. $r = 6 = p\sqrt{2} \Rightarrow p = \frac{6}{\sqrt{2}} = 3\sqrt{2}$
$q = p = 3\sqrt{2}$

23. The triangle is a 30-60-90 triangle, so the longer leg measures $5.4\sqrt{3}$.
Thus, $A = \frac{1}{2}(5.4)(5.4\sqrt{3}) \approx 25.3$ units^2.

30. One side $(2\sqrt{3})$ is twice the shortest side $(\sqrt{3})$ and the third side (3) is $\sqrt{3}$ times the shortest side, so this is a 30-60-90 triangle.

36. $P = 8 + 8 + 8 = 24$ units
To find the height, divide it into two 30-60-90 triangles. The smallest side is $\frac{8}{2} = 4$ units, so the height is $4\sqrt{3}$ units.
$A = \frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3}$ units^2

Lesson 5.6

12. $d = \sqrt{(-3-6)^2 + (-3-12)^2} = \sqrt{(-9)^2 + (-15)^2} = \sqrt{306} \approx 17.49$

18. The coordinates are: $E(-5, -3)$, $J(2, 2)$, $H(5, -4)$.
$EJ = \sqrt{(-5-2)^2 + (-3-2)^2} = \sqrt{(-7)^2 + (-5)^2} = \sqrt{74}$
$JH = \sqrt{(5-2)^2 + (-4-2)^2} = \sqrt{3^2 + (-6)^2} = \sqrt{9 + 36} = \sqrt{45}$
$EH = \sqrt{(5-(-5))^2 + (-4-(-3))^2} = \sqrt{10^2 + (-1)^2} = \sqrt{101}$
$P = \sqrt{74} + \sqrt{45} + \sqrt{101} \approx 25.36$ units

25. $a = \sqrt{(1-(-2))^2 + (5-2)^2} = \sqrt{3^2 + 3^2} = \sqrt{18}$
$b = \sqrt{(6-1)^2 + (0-5)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{50}$
$c = \sqrt{(6-(-2))^2 + (0-2)^2} = \sqrt{8^2 + (-2)^2} = \sqrt{68}$
$a^2 + b^2 = (\sqrt{18})^2 + (\sqrt{50})^2 = 18 + 50 = 68$, $c^2 = (\sqrt{68})^2 = 68$.
Since $a^2 + b^2 = c^2$, the triangle is a right triangle.

28. $s = \sqrt{(4-0)^2 + (-1-3)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{32}$.
The hypotenuse is $s\sqrt{2} = \sqrt{32} \cdot \sqrt{2} = \sqrt{64} = 8$ units.

31. The hypotenuse is the segment between (6, 0) and (0, 8). The midpoint of the hypotenuse is $M = \left(\frac{6+0}{2}, \frac{0+8}{2}\right) = (3, 4)$.

Let $A = (0, 0)$, $B = (6, 0)$, $C = (0, 8)$,

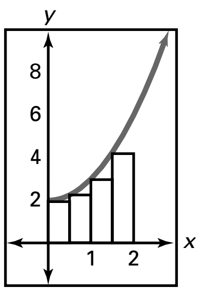
$AM = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

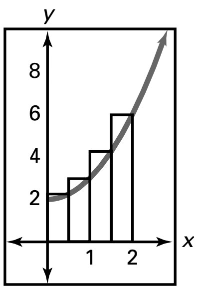
$BM = \sqrt{(6-3)^2 + (0-4)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

$CM = \sqrt{(3-0)^2 + (4-8)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

The distances are the same, 5 units.

37. Sample answer: Using rectangles of width 0.5, the height of each rectangle can be found by substituting the value of x into the equation $y = x^2 + 2$.





From left: $A = (0.5)(2) + (0.5)(2.25) + (0.5)(3) + (0.5)(4.25) = (0.5)(11.5)$
$= 5.75 \text{ units}^2 \rightarrow$ low estimate

From right: $A = (0.5)(2.25) + (0.5)(3) + (0.5)(4.25) + (0.5)(6) = (0.5)(15.5)$
$= 7.75 \text{ units}^2 \rightarrow$ high estimate

The average of the areas $= \frac{7.75 + 5.75}{2} = 6.75 \text{ units}^2$

If rectangles of width 1 are used, the estimate is about 7.

Lesson 5.7

13. I has same y-coordinate as H, so $I(x, q)$.

$HI = GJ = r$

Thus, the x-coordinate of I is r + x-coordinate of H, or $r + p$.

$I(r + p, q)$

18. $M(p, q)$; $N(r + s, q)$

22. Find the lengths of JM, KM, LM. If they are equal then the midpoint of the hypotenuse is equidistant from the three vertices.

$JM = \sqrt{\left(\frac{q}{2} - 0\right)^2 + \left(\frac{p}{2} - 0\right)^2} = \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{2}\right)^2}$

$KM = \sqrt{\left(\frac{q}{2} - 0\right)^2 + \left(\frac{p}{2} - p\right)^2} = \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{2}\right)^2}$

$LM = \sqrt{\left(\frac{q}{2} - q\right)^2 + \left(\frac{p}{2} - 0\right)^2} = \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{2}\right)^2}$

28. Let $ABCD$ be a square formed by:
$A(0, 0)$, $B(0, p)$, $C(p, p)$, and $D(p, 0)$
slope of diagonal $AC = \frac{p - 0}{p - 0} = 1$
slope of diagonal $BD = \frac{0 - p}{p - 0} = -1$
The two diagonals have slopes that are negative reciprocals of each other. Therefore they are perpendicular to each other.

Lesson 5.8

11. $P = \frac{\text{Shaded area}}{\text{Total area}} = \frac{\pi(1.5)^2 - \pi(0.5)^2}{\pi(1.5)^2}$
$= \frac{2.25\pi - 0.25\pi}{2.25\pi} = \frac{2\pi}{2.25\pi} = \frac{2}{2.25} = \frac{8}{9}$

14. $P = \frac{2}{3}$

16. $\frac{1}{4} = \frac{25}{100}$, or 25%

19. 50% means $\frac{500}{100} = 0.5$

26. $P = \frac{2}{5}$

37. $PB = \frac{\text{area of target } B}{\text{area of field}} = \frac{25 \cdot 25}{70 \cdot 100} = \frac{625}{7000} = \frac{5}{56}$
≈ 0.089