

Chapter 4 Parent Guide Congruent Triangles

Sometimes it is necessary to reproduce objects that are exactly the same size and shape to maintain uniformity in a man-made construction. Suppose, for example, you want to make a room whose walls are square. If one wall is a bigger square than the others, a gap would appear where the walls meet the ceiling or floor and the walls could not meet at right angles. This would affect shapes of adjacent rooms.

This course emphasizes congruence in triangles, because triangles are rigid. They cannot twist or lean out of shape. A rectangle made of four pieces of wood, for example, can “lean” unless the each corner is secured diagonally in some way, or unless another board connects sides to form a triangle. We depend upon triangles to strengthen our man-made constructions.

Besides proving triangles congruent, Chapter 4 proves some of the properties of quadrilaterals learned in Chapter 3. It also investigates congruencies in geometric constructions, which require only a compass and straightedge.

Lesson 4.1 develops an understanding of congruency by using various polygons. Lessons 4.2 through 4.4 focus on congruent triangles and corresponding parts of congruent triangles. Then Lessons 4.5 and 4.6 focus on how to use congruencies in special quadrilaterals. Lessons 4.7 and 4.8 use congruencies to construct figures by using only a straightedge and compass.

You can help your child understand the important concept of congruence in the activity below. You will need a ruler and protractor to complete this activity.

PROBLEM FOR DISCUSSION (See textbook page 210)

Two polygons are congruent if they are the same size and shape. Polygons 1 and 2 are congruent. If you slide one on top of the other, you will see that they match exactly. Can you think of a way to determine whether two polygons are congruent without actually moving them? What measurements would you need to make?

1. Discuss the meaning of congruence. If two quadrilaterals are congruent, what does that say about their sides and angles?

Congruence means that two objects have the exact size and shape.

If two quadrilaterals are congruent, then their sides have the same lengths and their angles have the same measures.

2. Are all pentagons congruent? Look at pentagon $KLMNO$ on page 211. Compare it to pentagon $PQRST$ on page 213. Are they the same size and shape?

No, not all pentagons are congruent because side lengths and angle measures may differ.

The pentagon on page 211 does not appear to have the same shape or angle measures as the pentagon on page 213.

3. Use a ruler to compare the lengths of the sides of pentagons $KLMNO$ and $PQRST$. Do the sides match? Use a compass to measure the angles of each pentagon. Do the angles match?

Using a ruler, you can determine that the length of ON is not the same as the length of RS . Therefore, their sides do not match.

Using a protractor, you can determine that $\angle S$ and $\angle R$ are not equal to the measures of $\angle N$ and $\angle O$. Therefore, their angles do not match.

4. Compare pentagon $PQRST$ on page 213 to pentagon $VWXYZ$ right below it. Are they the same size and shape? Measure the sides and angles. Do the sides match? Do the angles match?

Based on the labeling of each pentagon, the two pentagons have the same size and shape. Using a ruler and a protractor, you can confirm that the corresponding side lengths are the same and the corresponding angle measures are the same. Therefore, the two pentagons are congruent.

5. What have you learned about congruent polygons from this activity?

Congruent polygons have the same size and shape. Also, in order to prove that two polygons are congruent, you need to measure their sides and their angles.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 4.1

$$\begin{array}{ll}
 12. \angle QRS \cong \angle VWX & \angle TUQ \cong \angle YZV \\
 \angle RST \cong \angle WXY & \angle UQR \cong \angle ZVW \\
 \angle STU \cong \angle XYZ &
 \end{array}$$

16. No; the sides have different lengths.

$$\begin{array}{l}
 21. \text{ a. } \overline{ED} \\
 \text{ b. } \overline{DF} \\
 \text{ c. } \overline{EF}
 \end{array}$$

$$27. RP = RQ + QP = 3 + 5 = 8$$

$$\begin{array}{l}
 35. \text{ a. } \triangle RNO \cong \triangle AKC \\
 \overline{FRNT} \cong \overline{BAKX} \\
 \overline{FRAB} \cong \overline{TNKX} \\
 \overline{FBXT} \cong \overline{RAKN} \\
 \overline{RACO} \cong \overline{NKCO} \\
 \overline{FRONT} \cong \overline{BACKX}
 \end{array}$$

$$\begin{array}{l}
 \text{ b. Sample answer:} \\
 \angle ORN \cong \angle CAK \\
 \angle ONR \cong \angle CKA \\
 \angle RON \cong \angle ACK \\
 \angle RFT \cong \angle ABX \\
 \angle RFB \cong \angle NTX
 \end{array}$$

$$\begin{array}{l}
 \text{ c. Sample answer:} \\
 \overline{FR} \cong \overline{TN} \\
 \overline{BA} \cong \overline{XK} \\
 \overline{RA} \cong \overline{NK} \\
 \overline{FB} \cong \overline{TX} \\
 \overline{FT} \cong \overline{BX}
 \end{array}$$

Lesson 4.2

10. $\triangle LMN \cong \triangle PON$ by SAS, since $\angle MNL \cong \angle ONP$ because vertical angles are congruent.

17. Yes; ASA

30. By definition of a regular polygon, all the sides of the polygon are congruent. If a segment is drawn from the center to each vertex, the polygon is divided into triangles. By definition of the center of a regular polygon, the segments from the center to the vertices are all congruent, so the triangles are all congruent by SSS. Thus, the central angles are congruent because CPCTC.

38. No, AAA does not prove congruence.

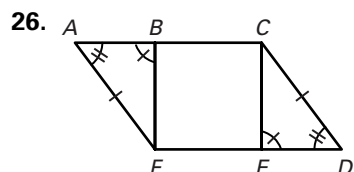
Chapter 4

Lesson 4.3

12. $\triangle HJK \cong \triangle MNL$; AAS

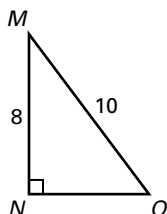
15. Can't be proven congruent

23. Reflexive Property of Congruence

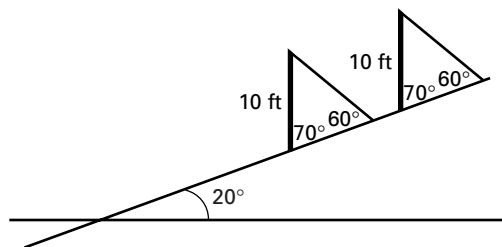


Statements	Reasons
$\angle 1 \cong \angle 4$	Given
$\overline{AF} \cong \overline{DC}$	
$\angle A \cong \angle D$	
$\triangle AFB \cong \triangle DCE$	AAS

33. Yes; HL



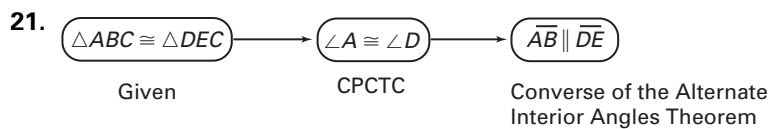
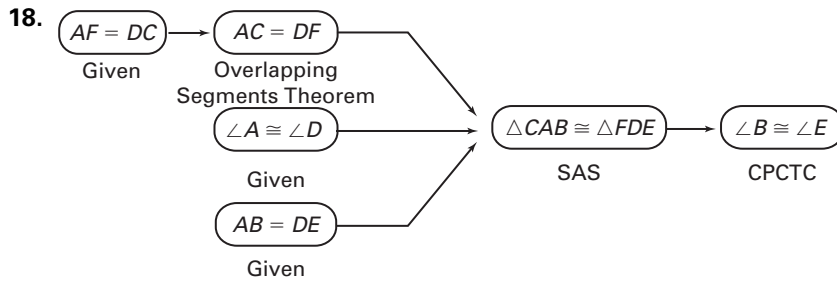
42.



The vertical poles make a 70° angle with the embankment. Assuming the wires are mounted to the poles at the same location, the wires must be the same length by AAS.

Lesson 4.4

15. By SAS $\triangle HFJ \cong \triangle GFJ$ and by CPCTC $\overline{GJ} \cong \overline{HJ}$
 thus, $GH = GJ + HJ = GJ + GJ = 12 + 12 = 24$.



32. Let ABC be an equilateral triangle. By the Isosceles Triangle Theorem $\angle B \cong \angle C$ because $\overline{AB} \cong \overline{AC}$ and $\angle A \cong \angle C$ because $\overline{BC} \cong \overline{BA}$. Hence $m\angle A = m\angle C = m\angle B$. By the Triangle Sum Theorem, $m\angle A + m\angle B + m\angle C = 180^\circ$. Using substitution, $m\angle A + m\angle A + m\angle A = 180^\circ$. Thus, $m\angle A = 60^\circ$ so $m\angle B = 60^\circ$ and $m\angle C = 60^\circ$.

Lesson 4.5

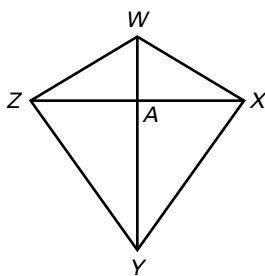
12. $m\angle D = 180^\circ - m\angle A = 180^\circ - 50^\circ = 130^\circ$

20. $m\angle E = m\angle C$
 $(2x + 6)^\circ = 50^\circ$
 $2x + 6 = 50$
 $2x = 44$
 $x = 22$
 $CD = FE = x - 7$
 $= 22 - 7$
 $= 15$

33. $\triangle ABD \cong \triangle CDB$

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68. Given: Kite $WXYZ$ with diagonals \overline{WY} and \overline{XZ} intersecting at point A
 Prove: $\overline{WY} \perp \overline{XZ}$



Statements	Reasons
$\overline{WX} \cong \overline{WZ}$	Given
$\overline{XY} \cong \overline{ZY}$	
$\overline{WY} \cong \overline{WY}$	Reflexive Property of Congruence
$\triangle WXY \cong \triangle WZY$	SSS
$\angle ZWY \cong \angle XWY$	CPCTC
$\overline{WA} \cong \overline{WA}$	Reflexive Property of Congruence
$\triangle WAX \cong \triangle WAZ$	SAS
$\angle WAX \cong \angle WAZ$	CPCTC
$m\angle WAX + m\angle WAZ = 180^\circ$	Linear Pair Property
$m\angle WAX + m\angle WAX = 180^\circ$	Substitution Property
$m\angle WAX = 90^\circ$	Division Property
$\overline{WY} \perp \overline{XZ}$	Definition of perpendicular

70. Theorem: A square is a rhombus.

A square is a quadrilateral with the property that all of its sides have the same length. This is the definition of a rhombus.

Lesson 4.6

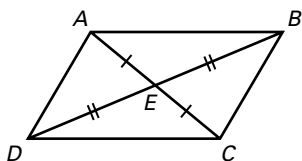
11. Yes, Theorem 4.6.3

15. Rhombus, Theorem 4.6.6

22. $KLMN$ is a rhombus by Theorem 4.6.7.
 $KLMN$ is a rectangle since $m\angle LKN = 90^\circ$.
 Therefore $KLMN$ is a square.

Chapter 4

40.



Given: Quadrilateral $ABCD$ and AC, BD bisect each other.

Prove: $ABCD$ is a parallelogram.

Statements	Reasons
\overline{AC} bisects \overline{BD} \overline{BD} bisects \overline{AC}	Given
$\overline{AE} \cong \overline{EC}$ $\overline{DE} \cong \overline{EB}$	Definition of bisector
$\angle AED \cong \angle BEC$	Vertical Angles Theorem
$\triangle AED \cong \triangle CEB$	SAS
$\angle EAD \cong \angle ECB$	CPCTC
$\overline{AD} \parallel \overline{BC}$	Alternate Interior Angles Theorem
$\angle AEB \cong \angle DEC$	Vertical Angles
$\triangle ABE \cong \triangle CDE$	SAS
$\angle EAB \cong \angle ECD$	CPCTC
$\overline{AB} \parallel \overline{CD}$	Converse of the Alternate Interior Angles Theorem
$ABCD$ is a parallelogram.	Definition of parallelogram

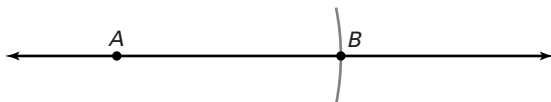
55. The boards that are 2 feet long must be opposite each other, and the boards that are 3 feet long must be opposite each other. She also should make sure that the diagonals have the same measure.

Lesson 4.7

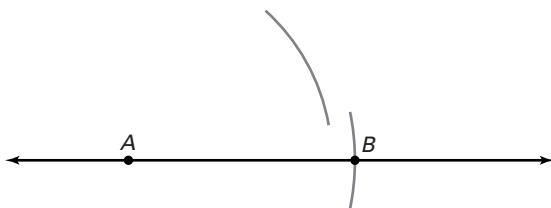
13. Draw a line and label point *A* on the line.



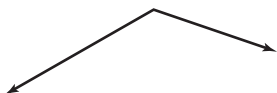
Set the compass to the distance of the bottom edge of the triangle. Place the point of the compass on *A* and draw an arc that intersects the line. Label the intersection point *B*.



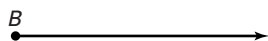
Set the compass to the distance of the left edge of the triangle. Place the point of the compass on *A* and draw an arc above *AB*.



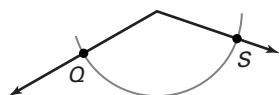
19. Trace the original angle.



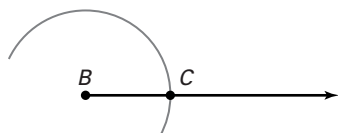
Use a straightedge to draw a ray with endpoint *B*.



Place the compass point on the vertex of the traced angle and draw an arc. Label the intersection points *Q* and *S*.

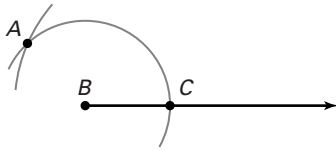


Without adjusting the compass, place the compass point at *B*. Draw an arc that crosses the ray. Label the intersection point *C*.

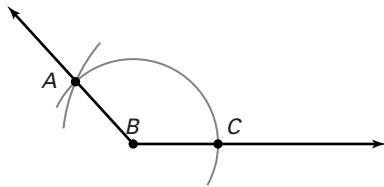


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Set the compass equal to the distance QS . Without adjusting the compass, place the compass at C and draw an arc that crosses the first arc. Label the intersection point A .



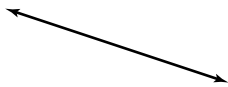
Draw \overrightarrow{BA} to form $\angle B$.



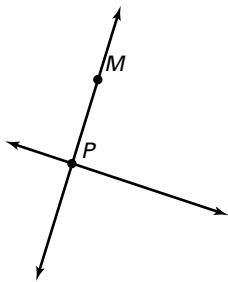
$\angle B$ is congruent to the given angle.

30. Trace the original figure. Label the point M .

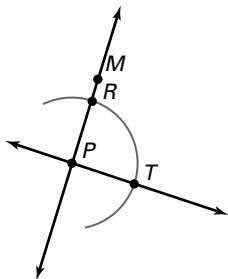
M



Draw a line through M that intersects the given line. Label the intersection point P .

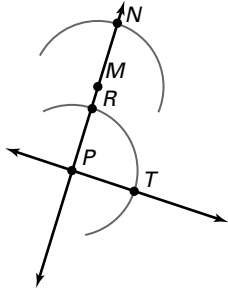


Draw an arc with center at P . Label new intersection points R and T .

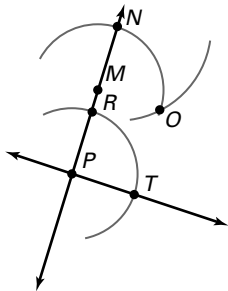


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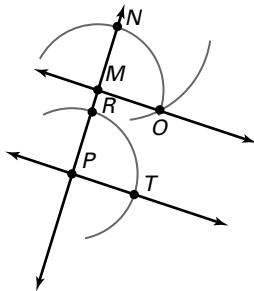
Without adjusting the compass setting, place the compass point at M and draw a new arc. Label the new intersection point N .



Set the compass to the distance RT . Place the compass point on N and draw an arc. Label the point of intersection O .



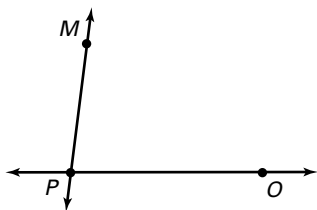
Draw the line containing M and O .



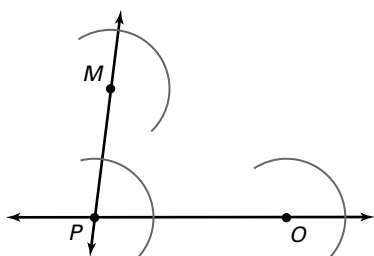
The line \overleftrightarrow{MO} is parallel to the given line.

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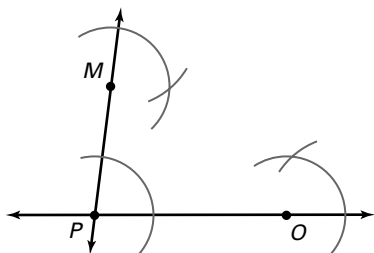
- 33.** Sample answer: A parallelogram has two points of parallel lines. Begin by drawing a pair of intersecting lines. Label the intersection point P and a point on each line M and O .



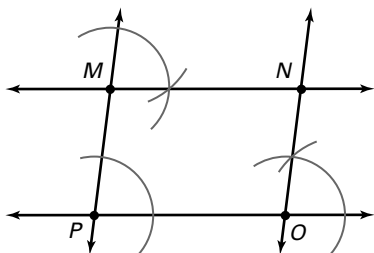
Using the same compass setting, make an arc with center at P , an arc with center at M , and an arc with center at O .



Set the compass to the distance between the intersection points of the arc at P . Use this setting to draw an arc from the intersection of the arc at M and the line \overleftrightarrow{PM} and to draw an arc from the intersection of the arc at O and the line \overleftrightarrow{PO} .



Draw the line containing M and the intersection of the two arcs near M and the line containing O and the intersection of the two arcs near O . Label the intersection point of these lines N .

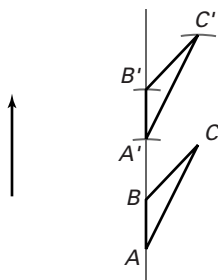


Quadrilateral $MNOP$ is a parallelogram.

43. Statements	Reasons
$\overline{AB} \cong \overline{AD}$	Same compass setting
$\overline{CB} \cong \overline{CD}$	
$ABCD$ is a kite.	Definition of kite
$\overline{AC} \perp \overline{BD}$	Diagonals of a kite are perpendicular.
$\overline{BD} \parallel \ell$	ℓ is an extension of line segment BD .
$\overline{AC} \perp \ell$	A line which is perpendicular to a line which is parallel to the given line is also perpendicular to the given line.

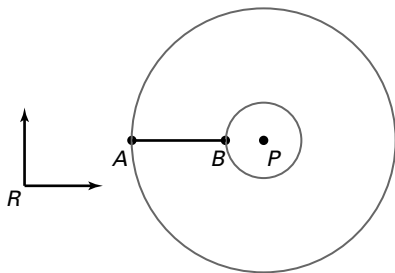
Lesson 4.8

10.

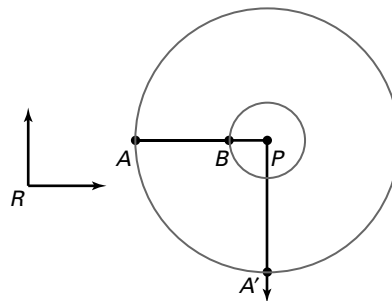


15. Follow the steps given.

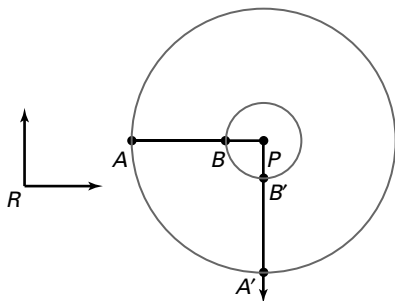
a.



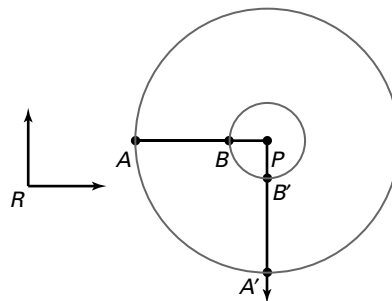
b.



c.



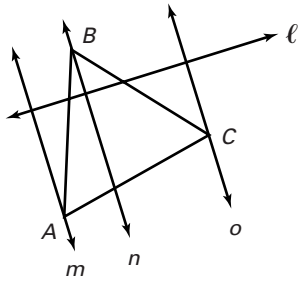
d.



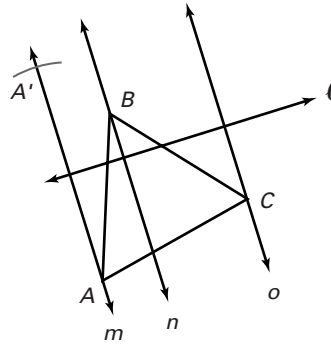
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19. Follow the steps given.

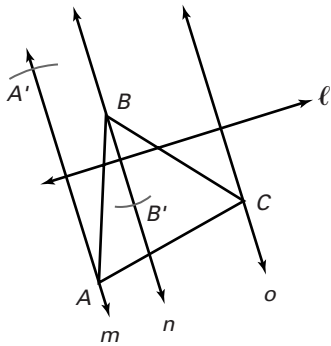
a.



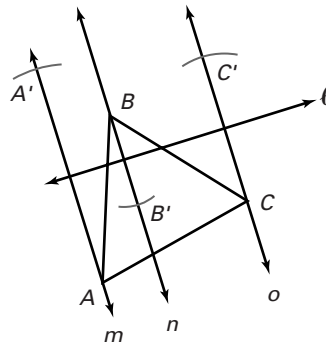
b.



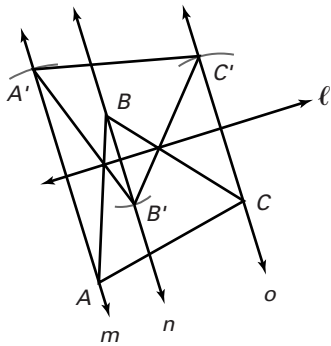
c.



d.

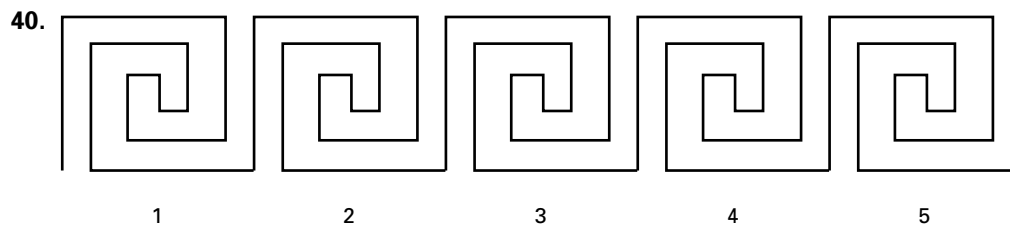


e.



23. $\triangle JKL$ is possible.

Chapter 4



Sample answer: a rotation of 90° about B

47.

Statements	Reasons
$\angle XZY \cong \angle YZS$	Vertical Angles Theorem
$\angle XQZ \cong \angle YZS$	Alternate Interior Angles Theorem
$\overline{QZ} \cong \overline{SZ}$	Diagonals of a parallelogram bisect each other.
$\triangle QXZ \cong \triangle SYZ$	ASA
$\overline{XZ} \cong \overline{YZ}$	CPCTC