

Chapter 3 Parent Guide 3 Parallels and Polygons

Topics in Chapter 3 will lead your child to a better understanding of how lines and segments work together to form angles and geometric polygons. A polygon is a plane figure having three or more segments as its sides. The chapter will place an emphasis on properties of triangles and quadrilaterals.

Most man-made structures are created with polygons. They are easier to measure, to reproduce, and to fit together than irregular shapes. Architects, electricians, and carpenters are among the many professionals who use polygons in their work. They need to know how the sides relate to one another. They also need to know the kind of angles various polygons can have.

Chapter 3 develops these concepts by showing how parallel lines can create reliable angles in polygons.

Among other things, it will also show how the number of sides of a polygon determines the sum of the angle measures.

Lesson 3.1 opens the chapter by examining symmetry in polygons. Lesson 3.2 studies special properties of quadrilaterals. Lesson 3.3 discusses parallel lines and the angles created by a transversal. In Lesson 3.4, your child will prove that pairs of lines are parallel. Lessons 3.5 and 3.6 address the sum of the interior angles of triangles and other polygons. Lesson 3.7 examines midsegments of triangles and trapezoids and the angles they form. Lesson 3.8 applies the ideas of this chapter to coordinate geometry.

You can help your child develop a clearer understanding of quadrilaterals by doing the following activity with him or her. It will show how special quadrilaterals compare to one another.

PROBLEM FOR DISCUSSION (See textbook page 148)

Any four-sided polygon is a quadrilateral. Quadrilaterals that have certain properties are called special quadrilaterals. Study the definitions and pictures of the quadrilaterals on page 148. Then discuss these questions.

1. What do all quadrilaterals have in common? How can they differ from one another?

All quadrilaterals have four sides. This can be easily remembered by its prefix, *quadra*, meaning four.

Quadrilaterals can differ from each other in a couple of ways:

- opposite sides may or may not be parallel
 - the number of congruent sides
 - the relationship between adjacent sides
2. Compare the trapezoid to the parallelogram. What would you have to change in the trapezoid to turn it into a parallelogram? What would you have to change in the parallelogram to turn it into a trapezoid?

To change a trapezoid into a parallelogram, you would have to make the

set of opposite sides that are not parallel, parallel.

To change a parallelogram into a trapezoid, you would have to change one set of opposite sides so they are not parallel.

The definition of a trapezoid states that *only* one set of opposite sides are parallel to each other, and a parallelogram has two sets of parallel sides.

3. Study the square. The properties of which two special quadrilaterals are combined to make a square? Explain why all squares are rectangles. Explain why all squares are rhombuses.

The properties of a rhombus and a rectangle are combined to make the square.

All rhombuses have 4 congruent sides and so does the square. This makes all squares rhombuses.

All rectangles have four 90° angles and so does the square. This makes all squares rectangles.

4. Choose one of the two parallelograms on this page. Describe what you would have to change to turn it into a rectangle.

In both of the parallelograms on this page, “straightening” out the parallelogram can form a rectangle. Since a rectangle needs four right angles, each of the four angles of the parallelogram need to be made into right angles.

5. What have you learned about quadrilaterals from this activity?

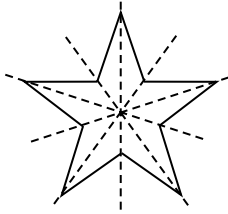
Quadrilaterals are four-sided polygons. The special types of quadrilaterals include trapezoids, parallelograms, rhombuses, rectangles, and squares.

The following are complete worked out solutions to selected exercises in the student textbook.

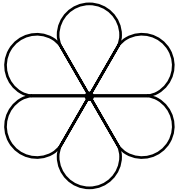
These solutions are provided to you so that you can help your child with their homework. Your child’s classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 3.1

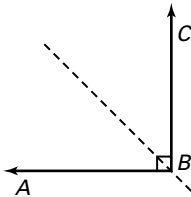
9.



20.

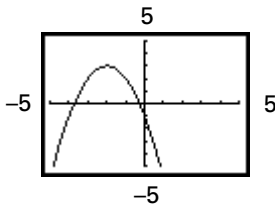


30. The axis of symmetry goes through the vertex of $\angle ABC$. The axis of symmetry is the angle bisector of $\angle ABC$.



34. Reflection of \overline{AD} across the horizontal axis of symmetry \overleftrightarrow{AC} or rotation of \overline{AD} through $m\angle BAD$ about A.

42.



Axis of symmetry: $x = -2$

56. The sign is a regular quadrilateral or square. It has 4 axes of symmetry—2 lines through midpoints of opposite sides and 2 lines through opposite vertices. It has nontrivial rotational symmetries for rotations of 90° , 180° , and 270° .

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Lesson 3.2

13. $m\angle WXY = m\angle WZY = 130^\circ$ because opposite angles of a parallelogram are congruent.
18. $FJ = \frac{1}{2}(FH) = \frac{1}{2}(15) = 7.5$ because the diagonals of a rhombus bisect each other.

34. Diagonals of a rectangle are congruent. Therefore,

$$\begin{aligned}x - 2 &= \sqrt{x} \\(x - 2)^2 &= (\sqrt{x})^2 \\x^2 - 4x + 4 &= x \\x^2 - 5x + 4 &= 0 \\(x - 4)(x - 1) &= 0\end{aligned}$$

Therefore $x = 4$ or $x = 1$. Since $x - 2$ is the length of a diagonal, $x = 1$ does not make sense because this would give a negative diagonal length. Thus, $x = 4$, so $WY = x - 2 = 4 - 2 = 2$, and $XZ = WY = 2$.

40. False. A square is both a rectangle and a rhombus.

Lesson 3.3

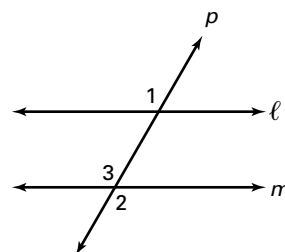
10. $\angle 3 \cong \angle 2$, because $\angle 2$ and $\angle 3$ are vertical angles. $\angle 6 \cong \angle 2$ because $\angle 2$ and $\angle 6$ are corresponding angles. $\angle 7 \cong \angle 2$ because $\angle 6$ and $\angle 7$ are vertical angles and $\angle 6$ and $\angle 2$ are corresponding angles.

14. $\angle 2 \cong \angle 5$ by the Vertical Angles Theorem.

25. $m\angle BDE = 25^\circ$ because it is an alternate interior angle to $\angle DBC$.

34. Given: Line $\ell \parallel$ line m . Line p is a transversal.
Prove: $\angle 1 \cong \angle 2$

Statement	Reasons
Line $\ell \parallel$ line m Line p is a transversal.	Given
$\angle 1 \cong \angle 3$	Corresponding Angles Postulate
$\angle 3 \cong \angle 2$	Vertical Angles Theorem
$\angle 1 \cong \angle 2$	Transitive Property of Congruence



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- 38.** They are congruent because they are alternate interior angles.
- 43.** $\angle 2$ and $\angle 3$ are congruent by the Alternate Interior Angles Theorem. By the given property of reflection, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, Thus $\angle 1 \cong \angle 4$ by the Transitive Property of Congruence.

Lesson 3.4

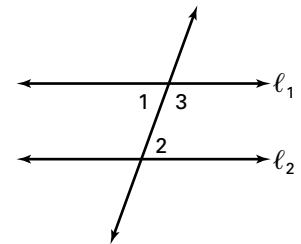
10. Alternate Interior Angles Theorem

- 16.** Given: $m\angle DFH = 55^\circ$; $m\angle GHF = 125^\circ$
 $\angle DFH$ and $\angle GHE$ are corresponding angles.

Statement	Reasons
$m\angle DFH = 55^\circ$; $m\angle GHF = 125^\circ$ $\angle DFH$ and $\angle GHE$ are corresponding angles.	Given
$m\angle FHG + m\angle GHE = 180^\circ$	Linear Pair Property
$125^\circ + m\angle GHE = 180^\circ$	Substitution Property
$m\angle GHE = 55^\circ$	Subtraction Property
$\angle DFH \cong \angle GHE$	Angle Congruence Postulate
$\overline{DF} \parallel \overline{GH}$	Converse of the Corresponding Angles Postulate

- 24.** Given: $\angle 1 \cong \angle 2$
 Prove: $\ell_1 \parallel \ell_2$

Statement	Reasons
$\angle 1 \cong \angle 2$	Given
$m\angle 1 = m\angle 2$	Angle Congruence Postulate
$m\angle 1 + m\angle 3 = 180^\circ$	Linear Pair Property
$m\angle 2 + m\angle 3 = 180^\circ$	Substitution Property
$\angle 2$ and $\angle 3$ are supplementary.	Definition of supplementary angles
$\ell_1 \parallel \ell_2$	Converse of the Same-Side Interior Angles Theorem



- 31.** $\angle 3$

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Lesson 3.5

9. $m\angle B = 180^\circ - 45^\circ - 90^\circ = 45^\circ$

16. $m\angle EDB + m\angle ADE = 180^\circ$ because they are a linear pair. Thus, $m\angle EDB = 180^\circ - 60^\circ = 120^\circ$.

25. $(4x^2 - 10) + (8x + 5) + (x^2 + 2x + 10) = 180$
 $5x^2 + 10x + 5 = 180$
 $5x^2 + 10x - 175 = 0$
 $x^2 + 2x - 35 = 0$
 $(x + 7)(x - 5) = 0$
 $x + 7 = 0 \quad x - 5 = 0$
 $x = -7 \quad \text{or} \quad x = 5$

Since angle measures are positive and

$m\angle E = 8x + 5$, x cannot equal -7 . Thus $x = 5$,

$m\angle D = [4(5)^2 - 10]^\circ = 90^\circ$,

$m\angle E = [8(5) + 5]^\circ = 45^\circ$, and

$m\angle F = [(5)^2 + 2(5) + 10]^\circ = 45^\circ$.

38. There are two possible exterior angles at each vertex. These angles can be seen by extending the sides of the triangle. Their measures are the same because both exterior angles are supplementary to the vertex angle.

Lesson 3.6

10. The sum of the exterior angle measures is 360° .
 $360^\circ - 132^\circ - 120^\circ = 108^\circ$

17. A regular dodecagon has 12 sides and 12 congruent exterior angles that must add to 360° . So each exterior angle has measure $\frac{360^\circ}{12} = 30^\circ$. Hence each interior angle has measure $180^\circ - 30^\circ = 150^\circ$.

26. $m\angle B = (2x)^\circ = (2 \cdot 36)^\circ = 72^\circ$

35. $m\angle K = (4x - 37)^\circ = [4(27) - 37]^\circ = 71^\circ$

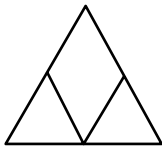
39. 3. Sample answer: a quadrilateral with angle measures $60^\circ, 70^\circ, 70^\circ, 160^\circ$ has 3 acute angles, but if a quadrilateral had 4 acute angles, then the sum of their measures would be less than 360° . Yes; for example, a rectangle has 4 right angles.

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Lesson 3.7

11. $IJ = \frac{1}{2}(50) = 25$
18. If parallel segments divide two sides of a triangle into four congruent segments, the length of the shortest segment is $\frac{1}{4}$ of the length of the base, the length of the middle segment is $\frac{2}{4}$ or $\frac{1}{2}$ of the length of the base and the length of the longest segment is $\frac{3}{4}$ of the length of the base.

24.



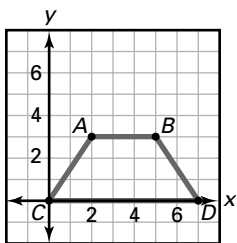
Rhombus; Two sides of the figure are triangle midsegments making opposite sides parallel, therefore a parallelogram is formed. Because the triangle is isosceles, the adjacent sides that are part of the triangle are congruent and half the length of the congruent sides of the triangle. The remaining adjacent sides are midsegments of the congruent triangle sides, and thus half of their length. So the lower sides are congruent to each other and the upper sides. Thus, the figure is a rhombus.

Lesson 3.8

14. $m = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$
 $MP = \left(\frac{-3 + 3}{2}, \frac{-1 + 3}{2} \right) = (0, 1)$
18. $m_1 = \frac{2 - 2}{3 - (-2)} = \frac{0}{5} = 0$ (horizontal);
 $m_2 = \frac{4 - (-1)}{2 - 2} = \frac{5}{0} = \text{undefined}$ (vertical)
Because all horizontal lines are perpendicular to all vertical lines, the segments are perpendicular.
25. $m_1 = \frac{-1 - 3}{3 - (-2)} = \frac{-4}{5}$,
 $m_2 = \frac{-1 - (-1)}{-2 - 3} = \frac{0}{-5} = 0$ (horizontal)
 $m_3 = \frac{-1 - 3}{-2 - (-2)} = \frac{-4}{0} = \text{undefined}$ (vertical)
Because one side is horizontal and another is vertical, it is a right triangle.

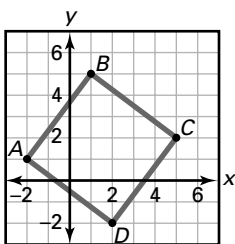
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30.



Trapezoid; The slope of \overline{CA} is $\frac{3}{2}$, the slope of \overline{BD} is $\frac{-3}{2}$, and the slopes of \overline{AB} and \overline{CD} are 0. Therefore \overline{AD} and \overline{CD} are parallel and the other pair is not. Because it has one set of parallel sides, $ABDC$ is a trapezoid.

35.



The slope of diagonal $\overline{AC} = \frac{(1-2)}{(-2-5)} = \frac{1}{7}$,
 the slope of diagonal $\overline{BD} = \frac{(5-(-2))}{(1-2)} = -7$.

$$m_{\overline{AC}} \cdot m_{\overline{BD}} = \frac{1}{7}(-7) = -1$$

Since the product of the two slopes is -1 , the diagonals are perpendicular.

42. $\overline{AB}: m = \frac{0-8}{2-0} = \frac{-8}{2} = -4, MP = \left(\frac{0+2}{2}, \frac{8+0}{2}\right) = (1, 4)$

$\overline{BC}: m = \frac{4-0}{3-2} = 4, MP = \left(\frac{2+3}{2}, \frac{0+4}{2}\right) = (2.5, 2)$

$\overline{AC}: m = \frac{4-8}{3-0} = \frac{-4}{3}, MP = \left(\frac{0+3}{2}, \frac{8+4}{2}\right) = (1.5, 6)$

Let $E = (1, 4)$, $F = (2.5, 2)$, $G = (1.5, 6)$ denote the midsegment endpoints.

Slope of $\overline{EF} = \frac{2-4}{2.5-1} = \frac{-2}{1.5} = \frac{-4}{3} = \text{slope of } \overline{AC}$

Slope of $\overline{EG} = \frac{6-4}{1.5-1} = \frac{2}{.5} = \frac{4}{1} = 4 = \text{slope of } \overline{BC}$

Slope of $\overline{FG} = \frac{6-2}{1.5-2.5} = \frac{4}{-1} = -4 = \text{slope of } \overline{AB}$

Thus each midsegment is parallel to a side of the triangle.