Reasoning in Geometry

Reasoning is a thinking process that progresses logically from one idea to another. Logical reasoning advances toward a conclusion in such a way as to be understood and accepted by others. Mathematical proofs are based on logical reasoning.

When a person's ideas are misunderstood, it is often because they were not presented logically. A listener may refuse to accept an excellent idea simply because a logical step was missed from the presentation.

Chapter 2 focuses on logical reasoning and proof. Proof is a convincing argument that uses logic to show that a statement is true. Proofs are used to communicate ideas logically from one person to another. The skill of writing a logical proof can transfer to the ability of communicating ideas so as to be accepted by the listener.

Geometry is a system of truths based on definitions and postulates. Conjectures are made from combining definitions and postulates and then proved or disproved. If the conjecture is proved, it becomes a theorem and can be used to prove or disprove other conjectures.

In Chapter 2, your child will learn the basics of how to write a logical proof. Lesson 2.1 discusses logical ways to analyze problems. Lesson 2.2 discusses the logic of if-then statements. Lesson 2.3 discusses the conditional statement of a definition. Lesson 2.4 builds a system on known algebraic properties and uses these properties and postulates in proofs. In Lesson 2.5, your child will make and prove or disprove conjectures.

Many people believe erroneously that geometry is the only place in mathematics that uses proof. Actually, proof is used in all mathematical disciplines. However, proof is usually learned in geometry because the concepts can be visualized. Conjectures come naturally from pictures.

You can help your child understand the concept of proof by doing the following activity with her or him.

PROBLEM FOR DISCUSSION (See textbook page 80)

Suppose that two squares are cut from opposite corners of a chessboard. Can the remaining squares be completely covered by 31 dominoes?

1. Discuss different ways of approaching the problem. Then decide which way is the most efficient way to solve the problem.

   You could take the problem literally and use an actual chessboard and actual dominoes to solve the problem.

   You could use the fact that one domino covers two chessboard squares, one of each color. Then expand on what would happen over the entire board.

   The latter is the probably the most efficient method.

2. Discuss exactly what you would say or do to convince someone that the remaining squares cannot be completely covered by 31 dominoes. Make sure your argument answers all possible questions that a listener might have.

   First, make sure that the person understands the way a chessboard looks. Every other square of a chessboard is the same color. Therefore, a light square is surrounded by four dark squares. Then explain that one domino covers two squares. In order to cover two squares, they must *not* be the same color. Any arrangement of the dominoes must cover the same number of dark squares as light squares. Most importantly, have the person notice that the squares that were cut off are the same color, leaving more squares of one color on the board than the other. Therefore, it is not possible to cover the altered board with the 31 dominoes.

3. Explain that the argument in Exercise 3 is a proof. Discuss the necessity of proof in mathematics and other fields.

   The argument in Exercise 3 is a proof because it explains why the remaining squares cannot be completely covered by 31 dominoes.

   Proofs are necessary in science and mathematics because in order to support a theory, you have to show that every step leading to the theory is supported by fact. If mathematics were based on feelings or something that could change from person to person, then there would be no uniformity to the field.

4. What have you learned about proofs from this activity?

   Proofs are needed to show that every step in a given theory, statement, etc. is true.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Chapter 2 - Solutions to Selected Exercises from the Student Textbook

Lesson 2.1

12. Sample answer: The expression on the left side of the equation represents multiplying the length by the width of the overall figure and thus is the area of that figure. The expression on the right side of the equation represents the sum of the areas of the pieces that form the overall figure and thus is the area of that figure. Therefore, the expressions on the left and right must be equal.

18. Column B contains entries for the number $2^3, 5^3, 8^3, \ldots$; while column C contains entries for the numbers $3^3, 6^3, 9^3, \ldots$

22. The next four terms are $\frac{1}{16 \cdot 2} = \frac{1}{32}; \frac{1}{32 \cdot 2} = \frac{1}{64}$
$\frac{1}{64 \cdot 2} = \frac{1}{128}; \frac{1}{128 \cdot 2} = \frac{1}{256}$

31. A square array of dots with n rows of n dots represents the number n^2. When the square is increased to $n + 1$ rows of $n + 1$ dots $n + n + 1$ dots will have been added to the number of dots in the original square.

34. 21

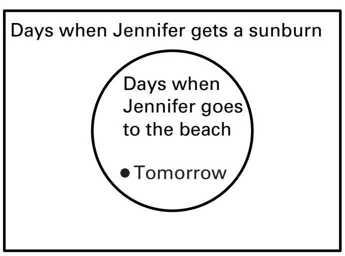
Lesson 2.2

14. If two angles form linear pairs, then the angles are supplementary angles.

18. Hypothesis: two angles are complementary
Conclusion: the sum of the angle measures is 90°
Converse: If the sum of the measures of two angles is 90°, then the angles are complementary.
The converse is true.

22. If $\angle AXB$ and $\angle BXD$ are supplementary, then $m\angle AXB + m\angle BXD = 180°$.

26. Conclusion: Jennifer will get a sunburn.



32. If ruskers bleer, then homblers frain.
If homblers frain, then quompies plaun.
If quompies plaun, then romples gleer.
Conclusion: If ruskers bleer, then romples gleer.

Lesson 2.3

9. **a.** Conditional: If a person is a teenager, then the person is from 13 to 19 years old.

b. Converse: If a person is from 13 to 19 years old, then the person is a teenager.

c. Biconditional: A person is a teenager if and only if the person is from 13 to 19 years old.

d. The statement is a definition because the conditional and the converse are both true.

19. They do not share a common side.

25. Shapes *b* and *c* are parallelograms because they are quadrilaterals with both pairs of opposite sides parallel. *a* is a triangle, not a quadrilateral, while *d* has only one pair of opposite sides parallel.

Lesson 2.4

10. Addition; Division

16. $m\angle MLN + m\angle NLP = m\angle PLQ + m\angle NLP$

21. $XZ = 15$ (by the Overlapping Segments Theorem)

26. $m\angle GHI$

29. Yes; Transitive Property

Lesson 2.5

14. $\angle ABC$ is supplementary to $\angle EBA$, so
$m\angle ABC = 180° - m\angle EBA = 180° - 112° = 68°$.

20.
$$\begin{aligned} 2(3x + 5) + 10x - 6 &= 180 \\ 6x + 10 + 10x - 6 &= 180 \\ 16x + 4 &= 180 \\ 16x &= 176 \\ x &= 11 \end{aligned}$$

$$\begin{aligned} m\angle ABC &= m\angle ABP + m\angle PBC \\ &= m\angle RBQ + m\angle QBC \\ &= [10(11) - 6]° + [3(11) + 5]° = 142° \end{aligned}$$

22. Deductive reasoning. The argument is a proof. Because the hypothesis occurred, the conclusion is proved.

30. Double the distance between reflecting lines is $2(10 \text{ cm}) = 20 \text{ cm}$.

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