

Algebra Essentials and Applications Internet Activity

ME1 Continued Fractions Investigations

A continued fraction might take the following form:

$$a + \frac{1}{b + \frac{1}{c + \frac{1}{d + \frac{1}{e + \dots}}}}$$

where the letters a , b , c , d , and e represent numbers. (The “...” means “continue in the same fashion.”) Such fractions are useful and important in many areas.

1. Some continued fractions are simple and easy to follow. This one represents $\sqrt{2}$:

$$1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

Use this fraction and a calculator to approximate the value of $\sqrt{2}$. To six decimal places, $\sqrt{2} \approx 1.414214$. Were you close?

2. This is the continued fraction for $\sqrt{10}$:

$$3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \dots}}}}$$

Use this fraction and a calculator to approximate the value of $\sqrt{10}$. To six decimal places, $\sqrt{10} \approx 3.162278$. Were you close?

3. Here is a continued fraction for a familiar constant:

$$3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{292 + \dots}}}}$$

Use this fraction and a calculator to approximate the value of this constant. What is the constant represented by this continued fraction?

4. To see how a continued fraction can be used to approximate an irrational number, return to the continued fraction for $\sqrt{2}$:

$$1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

Start at the last term shown and, working from the bottom up, re-write the entire expression as a single fraction:

$$\begin{aligned} 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}} &= 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\frac{5}{2}}}} \\ &= 1 + \frac{1}{2 + \frac{1}{2 + \frac{2}{5}}} \\ &= 1 + \frac{1}{2 + \frac{1}{\frac{12}{5}}} \\ &= 1 + \frac{1}{2 + \frac{5}{12}} \\ &= 1 + \frac{1}{\frac{29}{12}} \\ &= 1 + \frac{12}{29} \\ &= \frac{41}{29} \end{aligned}$$

Use a calculator to approximate $\sqrt{2}$ with this fraction. How good is the approximation? To get a better approximation, we would have to use more terms, making this continued fraction longer or “deeper.”
