

What We Are Learning

Ratios, Rates, and Proportions

Vocabulary

These are the math words we are learning:

conversion factor

a fraction whose numerator and denominator represent the same quantity but use different units

cross product in a proportion, the product of a numerator on one side with the denominator on the other

equivalent ratio

ratios that name the same comparison

proportion an equation that states that two ratios are equivalent

rate a ratio that compares two quantities measured in different units

ratio a comparison of two quantities by division

unit price a unit rate used to compare prices

unit rate a rate in which the second quantity in the comparison is one unit

Dear Family,

The student will be learning about ratios. A **ratio** is simply a comparison of two items by division. Ratios are often expressed as fractions.

The student will use ratios that name the same comparison as another ratio. These ratios are called **equivalent ratios**. One way the student can determine if two ratios are equivalent, or in **proportion**, is if the ratios can be simplified to the same value.

Simplify to tell whether the ratios form a proportion.

$$\frac{10}{15} \text{ and } \frac{24}{40}$$

$$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

$$\frac{24}{40} = \frac{24 \div 8}{40 \div 8} = \frac{3}{5}$$

Since, $\frac{2}{3} \neq \frac{3}{5}$, the ratios are not in proportion.

Another way to determine if two ratios are proportional is to find the cross products of two ratios. If the **cross products** of the ratios are equal, the ratios are in proportion. If the cross products are not equal, the ratios are NOT in proportion.

Tell whether the ratios are proportional.

$$\frac{4}{12} \stackrel{?}{=} \frac{3}{9}$$

$$\frac{4}{12} \swarrow \searrow \frac{3}{9}$$

Find the cross products.

$$36 = 36$$

Since the cross products are equal, the ratios are proportional.

The student will apply the properties of proportions to help with problems that involve ratios. When one of the values of a proportion is unknown, the student will learn to solve for that missing value by using cross products or properties of equality.

The student will also use proportions and conversion factors to solve problems involving **rates**.

The student may use some of these common conversions when he or she needs to convert units in order to find a solution.

Measure	Customary System	Metric System
Length and Distance	12 in. = 1 ft 3 ft = 1 yd 5280 ft = 1 mi	10 mm = 1 cm 100 cm = 1 m 1000 m = 1 km
Volume and Capacity	2 cups = 1 pt 2 pints = 1 qt 4 qt = 1 gal	1000 mL = 1 L
Weight and Mass	16 oz = 1 lb 2000 lb = 1 ton	1000 mg = 1 g 1000 g = 1 kg

Mary is filling up the sand box with 8 bags of sand. Each bag weighs 3 lb. Use conversion factors to find how many ounces of sand are in each bag.

The problem gives the ratio 3 lb to 1 bag and asks for the answer in *ounces* per bag.

$$\frac{3 \text{ lb}}{1 \text{ bag}} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} \quad \text{Multiply by the conversion factor.}$$

$$= \frac{3 \cdot 16 \text{ oz}}{1 \text{ bag}} \quad \text{Cancel the lb units.}$$

$$= 48 \text{ oz per bag}$$

Have the student solve proportions that involve real life information. This will allow the student to see the application of this concept outside the classroom.

Sincerely,

CHAPTER
5 **At-Home Practice**
Ratios, Rates, and Proportions

Simplify each ratio.

1. $\frac{6}{8}$

2. $\frac{12}{9}$

3. $\frac{20}{30}$

Simplify to tell whether the ratios are equal.

4. $\frac{12}{15}$ and $\frac{28}{35}$

5. $\frac{12}{30}$ and $\frac{30}{75}$

6. $\frac{45}{63}$ and $\frac{36}{96}$

Determine the lower unit price.

7. 6 cans of soup for \$3.39
or 4 cans of soup for \$2.29

8. 28-oz bottle of juice for \$2.29
or 64-oz bottle of juice for \$5.59

Find the appropriate factor for each conversion.

9. It took Sam 40 seconds to run the 160 yards from his house to Steve's house. Find Sam's average speed in yards per second. Use dimensional analysis to check the reasonableness of your answer.

10. A truck traveled 550 ft down a road in 11 seconds. How many miles per hour was the truck traveling?

Tell whether each pair of ratios is proportional.

11. $\frac{15}{24}$ and $\frac{5}{8}$

12. $\frac{9}{7}$ and $\frac{11}{19}$

13. $\frac{14}{16}$ and $\frac{35}{40}$

Solve each proportion.

14. $\frac{m}{9} = \frac{16}{12}$

15. $\frac{12}{7} = \frac{42}{x}$

16. $\frac{3}{5} = \frac{d}{60}$

Answers: 1. $\frac{4}{3}$ 2. $\frac{3}{2}$ 3. $\frac{3}{2}$ 4. The ratios are in proportion. 5. The ratios are in proportion. 6. The ratios are not in proportion. 7. 6 cans for \$3.39 8. 28 oz for \$2.29 9. 4 yd/s; 8.2 mi/h is reasonable 10. 34 mi/h 11. yes 12. no 13. yes 14. $m = 12$ 15. $x = 24$ 16. $d = 36$

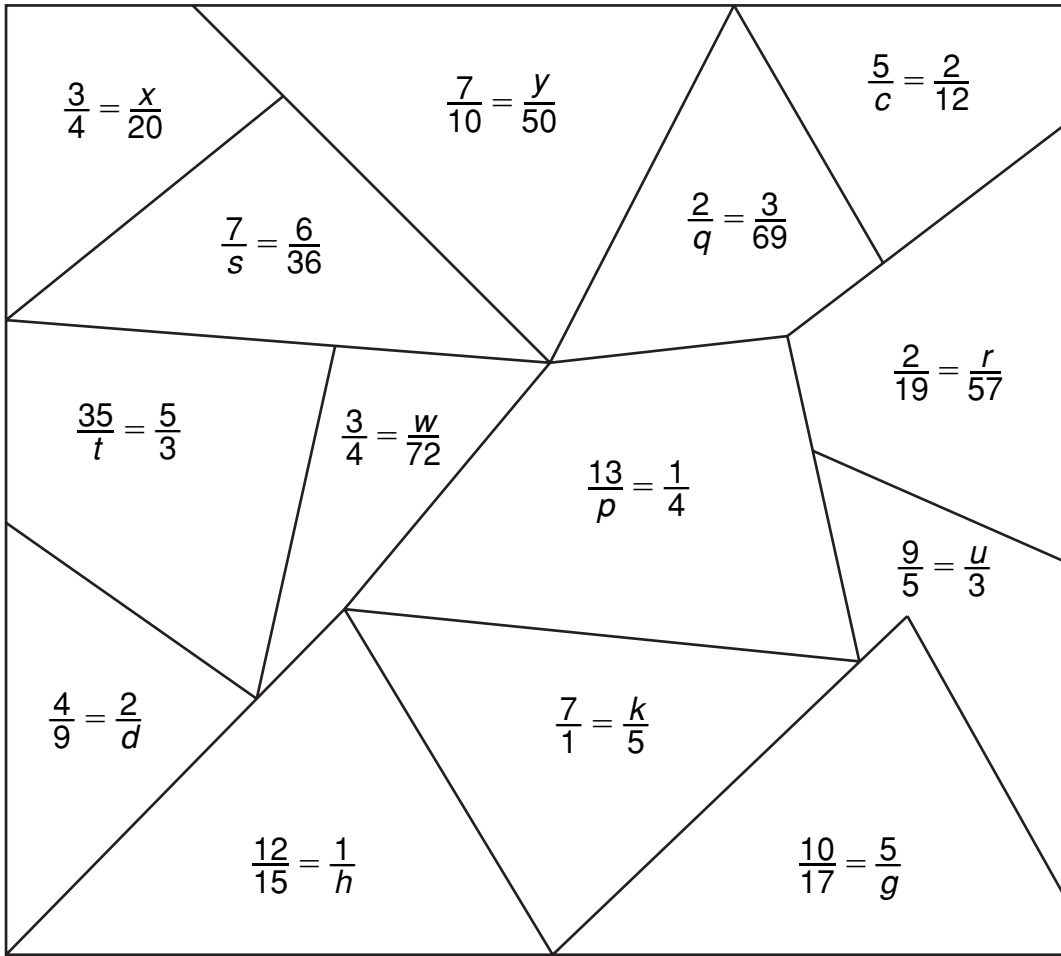
CHAPTER
5 **Family Fun**
Proportional Patterns

Materials

Red and blue markers

Directions

Solve the proportions. If the last digit of the unknown value is even, color that particular area blue. If the last digit of the answer is odd, color that particular area red.



Answers: $c = 30, x = 15, y = 15, q = 35, p = 46, t = 21, w = 54, p = 54, r = 52, r = 6, d = 4.5, s = 42, g = 8.5, k = 35, h = 1.25, u = 5.4$

What We Are Learning

Similarity and Scale

Vocabulary

These are the math words we are learning:

corresponding angles

matching angles of two or more polygons

corresponding sides

matching sides of two or more polygons

indirect measurement

a method of using similar figures and proportions to find a measure

similar figures that have the same shape, but not necessarily the same size

scale the ratio between two sets of measurements

scale drawing a drawing that uses a scale to make an object smaller than or larger than the real object

scale factor the ratio used to enlarge or reduce similar figures

scale model a three-dimensional model that accurately represents a solid object

Dear Family,

The student will be studying similar figures. **Similar figures** are figures that are the same shape, but not necessarily the same size. However, the angles of the figures do have to be congruent, and the ratios of corresponding sides must be equivalent. Since the sides of similar figures are proportional, you can find an unknown dimension by using the properties of proportions.

Sam needs to pack a jewelry box that measures 4.5 inches wide and 6 inches long. If he finds a box that is similar with a length of 10 inches, how wide is the box?

$$\frac{4.5 \text{ in.}}{6 \text{ in.}} = \frac{x \text{ in.}}{10 \text{ in.}}$$

$$6 \cdot x = 4.5 \cdot 10$$

$$6x = 45$$

$$x = \frac{45}{6} = 7.5$$

Set up a proportion.

Find the cross products.

Multiply.

Solve for x .

The width of the box is 7.5 inches.

When an item is too large to view on paper, you need to make a scale drawing or scale model of the item. A **scale drawing** is an accurate two-dimensional representation of an object. A **scale model** is a three-dimensional representation of the actual object. Both the scale model and drawing are similar to the actual object.

A **scale** is used to show the ratio between the dimensions of the scale drawing or model and the actual object. This ratio is the **scale factor**. The student will learn to identify and use the scale to find the dimensions of a scale drawing, model, or actual object.

The length of an object on a scale drawing is 4 cm and its actual length is 400 m.

The scale is 1 cm: _____ m. What is the scale?

$$\frac{1 \text{ cm}}{x \text{ m}} = \frac{4 \text{ cm}}{400 \text{ m}}$$

$$400 \cdot 1 = 4 \cdot x$$

$$x = 100$$

Set up a proportion: $\frac{\text{scale length}}{\text{actual length}}$

Find the cross products.

Solve the proportion.

The scale is 1 cm:100 m.

You can use scale factor and similar figures to find missing measurements of similar figures. This is called **indirect measurement**.

A model of a 16-foot boat was made using the scale 3 in.:4 ft. What is the height of the model?

$$\frac{3 \text{ in.}}{4 \text{ ft}} = \frac{3 \text{ in.}}{48 \text{ in.}} = \frac{1 \text{ in.}}{16 \text{ in.}} \quad \text{First find the scale factor.}$$

Now that you have the scale factor, you can set up the proportion.

$$\begin{array}{ll} \frac{1}{16} = \frac{h \text{ in.}}{192 \text{ in.}} & \text{Convert: } 16 \text{ ft} = 192 \text{ in.} \\ 16h = 192 & \text{Cross multiply.} \\ h = 12 & \text{Solve for the height.} \end{array}$$

The height of the model is 12 in.

The material in this section has many real-life applications. Have the student explain how to use a scale factor in relation to models and scale drawings. Practice having the student convert model dimensions to actual dimensions.

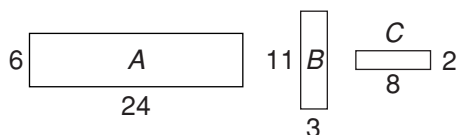
Sincerely,

CHAPTER
5 **At-Home Practice**
Similarity and Scale

Use the properties of similar figures and indirect measurement to answer each question.

1. A 5 in. long by 7 in. wide picture is going to be made into a similar poster with a length of 6 ft. How wide will the poster be?

2. Is rectangle A similar to rectangle B or to rectangle C?



3. Using a scale of $\frac{1}{2}$ cm:2 m, how long is an object that measures 3.5 cm long in a scale drawing?

4. Using a 100x magnification microscope, a paramecium has a length of 2.8 mm. What is the actual length of the paramecium?

A cube with side lengths of 8 cm is built from small unit cubes. Compare the following values:

5. the side lengths of the two cubes

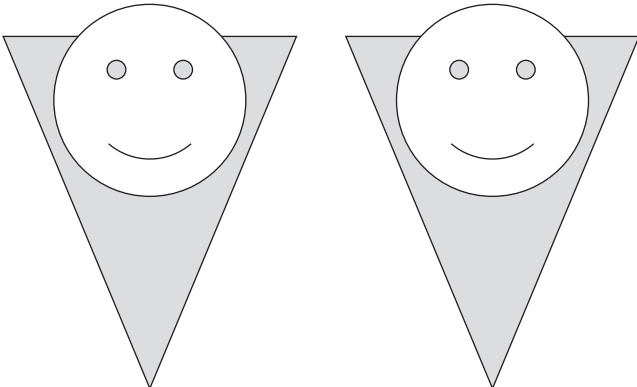
6. the surface areas of the two cubes

7. the volumes of the two cubes

Answers: 1. 8.4 ft 2. rectangle C 3. 14 m 4. 0.028 mm 5. The sides of the larger cube are 8 times longer than the smaller cube. 6. The surface area of the larger cube is 64 times that of the smaller cube. 7. The volume of the larger cube is 512 times that of the smaller cube.

CHAPTER
5 **Family Fun**
Scrambled Fun

Why do Tommy Triangle's twin brothers love math class?



Directions

Unscramble each of the clue words.

Take the letters that appear in boxes and unscramble them to get the answer to the riddle.

IOTAR

ROOPITRNPO

TERA

ROCSS RUDPOCT

NDRSIRCOENPOG

SLGNEA

SECLA RFTCAO

SOGIPENDNRORC

DESI

CELSA LEMDO

CYPCIATA

Answer: ratio, proportion, rate, cross product, corresponding angles, scale factor, corresponding sides, scale model, capacity; Because they are so similar.