CHAPTER 9

Family Letter

Section A

What We Are Learning

Quadratic Functions

VOCABULARY
These are the math words we are learning:

axis of symmetry
a line that divides a parabola into two symmetrical halves

minimum value
the least possible *y*-value of a function

minimum value
the greatest possible *y*-value of a function

quadratic function
a function that can be written in the form $y = ax^2 + bx + c$ where *a*, *b*, and *c* are real numbers

parabola
the type of curve represented by the graph of a quadratic function

vertex
the highest or lowest point on a parabola

zero of a function
an *x*-value that makes the function equal to zero

Dear Family,

In this section, students will identify and graph **quadratic functions**. Quadratic functions are very different than linear functions, but many of the same skills and processes can be applied to quadratic functions.

Students already know that a linear function is a function that can be written in the form $y = mx + b$, where *m* and *b* are real numbers. The graph of a linear function is a straight line.

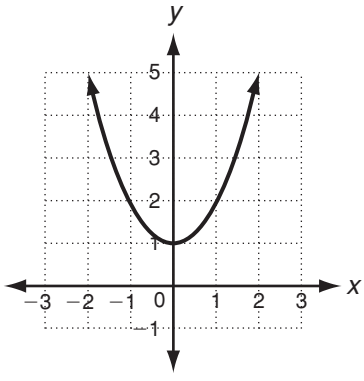
A **quadratic function** is a function that can be written in the form $y = ax^2 + bx + c$, where *a*, *b*, and *c* are real numbers and $a \neq 0$. The graph of a quadratic equation is a **parabola**, or a U-shaped curve.

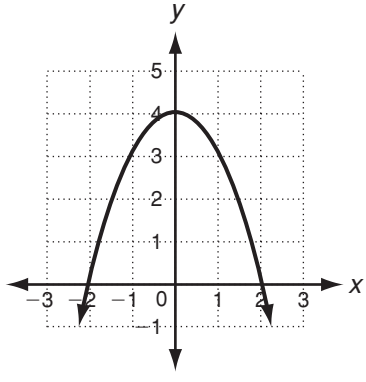
The graph of a quadratic function may open upward or downward depending on the value of the coefficient of the x^2-term.

GRAPHING HINTS:

Given the function $y = ax^2 + bx + c$,

if $a > 0$ the parabola opens upward (smiles).	if $a < 0$ the parabola opens downward (frowns).





$y = x^2 + 1$ means $y = \mathbf{1}x^2 + 1$ *a* is positive so the parabola is smiling.	$y = -x^2 + 4$ means $y = \mathbf{-1}x^2 + 4$ *a* is negative so the parabola is frowning.

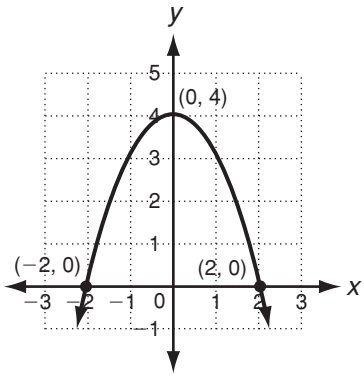
Family Letter

Section A continued

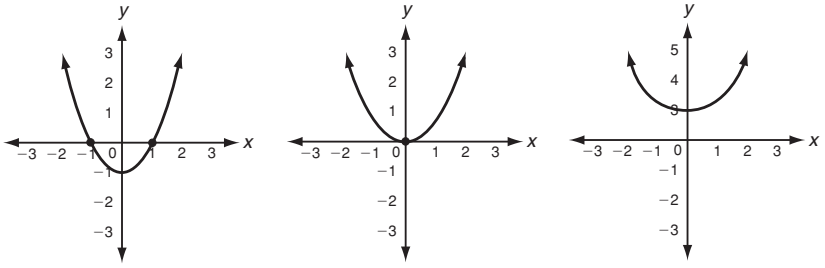
Vertex: The highest or lowest point on a parabola. In this graph, the vertex is (0, 4).

Maximum Value: The greatest *y*-value for the function. Here the maximum value is 4.

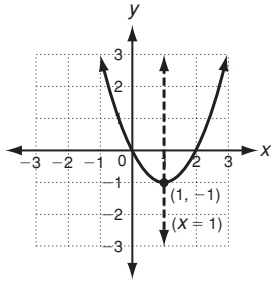
Zeros of a Function: *x*-values that makes the function equal to zero. The zeros occur where the parabola crosses the *x*-axis. The zeros are −2 and 2.



A quadratic function can have 2 zeros, 1 zero, or no zeros.



Axis of symmetry: the line that divides the parabola into two symmetrical halves at the minimum or maximum point. The axis of symmetry is $x = 1$.



Follow the Examples in the text with the student that show how to find the zeros of a quadratic function, the vertex, and the axis of symmetry.

To graph quadratic functions, students will follow these steps:

- Find the axis of symmetry by finding zeros or by using the formula $x = -\frac{b}{2a}$.
- Find the vertex and the *y*-intercept.
- Find two more points on the same side of the axis of symmetry as the point containing the *y*-intercept.
- Graph the axis of symmetry, the vertex, the point containing the *y*-intercept, and the two other points.
- Reflect those points across the axis of symmetry and connect with a smooth curve.

Sincerely,

Name _____ Date _____ Class _____

CHAPTER 9

At-Home Practice

Quadratic Functions

Without graphing, tell whether each point is on the graph of the given equation.

1. $y + 4x^2 = 41$; (3, 5)

2. $x^2 + y = 8$; (3, −2)

3. $y = 5x^2 - 9$; (2, 11)

_____ _____ _____

Tell whether the graph of each quadratic function opens upward or downward and whether the parabola has a maximum or minimum.

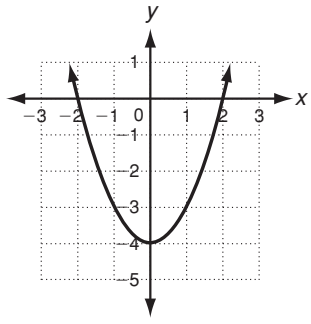
4. $y = 7x^2 + 8$

5. $y - 5x^2 = 6x + 2$

6. $4x^2 + y = 9x - 5$

_____ _____ _____

7. Find the zeros and axis of symmetry of the quadratic function.



_____ _____

Find the vertex of each parabola.

8. $y = x^2 - 4x + 5$

9. $y = -3x^2 - 6x + 4$

10. $y = x^2 - 5$

_____ _____ _____

Match the graph to the correct quadratic equation.

11. $y = 2x^2 + 1$

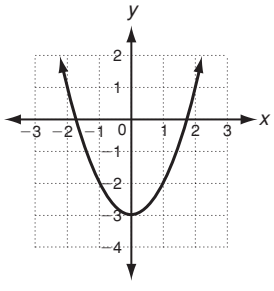
12. $y = -x^2 + 2$

13. $y = x^2 - 3$

14. $y = -2x^2 + x + 1$

_____ _____ _____ _____

A



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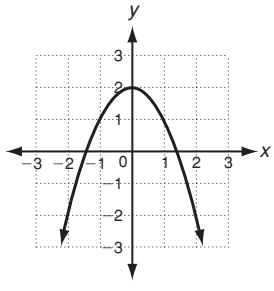

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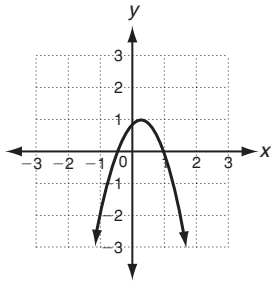

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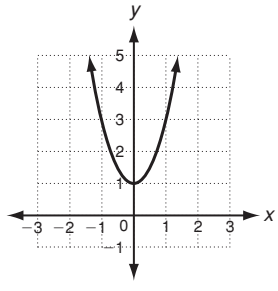


Answers: 1. yes; **2.** no; **3.** yes; **4.** upward and minimum; **5.** upward and minimum; **6.** downward and maximum; **7.** 2 and −2; $x = 0$; **8.** (2, 1); **9.** (−1, 7); **10.** (0, −5); **11.** D; **12.** B; **13.** A; **14.** C

Name _____ Date _____ Class _____

CHAPTER 9

Family Fun

Kitty Riddle

Objective: To practice finding the axis of symmetry of a quadratic equation.

Materials: puzzle sheet
pen or pencil

Directions:

- Find the axis of symmetry for each quadratic equation.
- Complete the table.
- Match the letter for each equation to the correct number below to answer the riddle.

Equation	Letter	Axis of Symmetry
$y = 6x^2 - 8x$	**F**	$x =$
$y = x^2 - 24x + 1$	**I**	$x =$
$y = -\frac{1}{2}x^2 - 4x - 2$	**B**	$x =$
$y = -2x^2 + 4x + 1$	**L**	$x =$
$y = \frac{1}{2}x^2 - 6x + 3$	**A**	$x =$
$y = -2x^2 + 10$	**R**	$x =$
$y = -x^2 + 12x + 3$	**E**	$x =$
$y = \frac{1}{4}x^2 + 6x + 10$	**O**	$x =$
$y = 3x^2 - 4x - 8$	**P**	$x =$
$y = 6x^2 - 48x - 8$	**A**	$x =$
$y = 2x^2 + 8x + 12$	**S**	$x =$
$y = 0.5x^2 - 2x - 0.75$	**N**	$x =$
$y = 6.25x^2$	**R**	$x =$
$y = -18 - 12x - 6x^2$	**U**	$x =$

____	____	____	____	–	____	____	____	____	____	____
$\frac{2}{3}$	-1	0	0		4	-4	-12	1	6	-2

Answers: **F.** $-\frac{2}{3}$; **I.** 12; **B.** -4; **L.** 1; **A.** 6; **R.** 0; **E.** 6; **O.** -12; **P.** $\frac{2}{3}$; **A.** 6; **S.** -2; **N.** 4; **R.** 0; **U.** -1

CHAPTER 9

Family Letter

Section B

What We Are Learning

Solving Quadratic Equations

VOCABULARY
These are the math words we are learning:

quadratic equation
an equation that can be written in the standard form of $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$

completing the square
to add a term to $x^2 + bx$ to form a perfect square trinomial

discriminant
the value $b^2 - 4ac$

Dear Family,

In this section students will solve quadratic equations. Just as for a linear equation, the solution of a quadratic equation is an ordered pair.

Some solving methods are better suited for some equations than others, so students will learn five solution methods, all of which use previously mastered skills. By reading the text Examples for each method and completing the homework, the student will become proficient in choosing and using methods for solving quadratic equations.

Listed below is a brief description of each method and some tips that can be used to guide students through this section.

METHOD 1: SOLVE BY GRAPHING

- Write the related function by rewriting the function by replacing 0 with y.
- Graph the function.
- Find the zeros of the function. Remember the function may have 2, 1, or no zeros.
- Note: There are benefits in using a graphing calculator to graph quadratic functions.

METHOD 2: SOLVE BY FACTORING

- When using the Zero Product Property, the factored polynomial *must* equal zero.
- Factor completely.
- Look for special products like difference of two squares or perfect-square trinomials.
- Solutions are found by setting each factor equal to 0. For example, $(x - 5)(x + 2) = 0$ has solutions 5 and -2.
- Use the FOIL method to check your factoring.

METHOD 3: SOLVE BY USING SQUARE ROOTS

- Use the formula $x^2 = a$.
- Take the square root of each side.
- Find both the positive and negative square root. The symbol for this is $\pm\sqrt{\ }$.
- Check your answers by substituting the solution into the original equation.

METHOD 4: COMPLETE THE SQUARE

- Create a perfect-square trinomial by dividing the coefficient of the x-term by 2 and then squaring that quotient to get the constant term.
- Write the given equation in the form $x^2 + bx = c$.
- Find $\left(\frac{b}{2}\right)^2$.
- Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
- Factor the perfect-square trinomial.
- Take the square roots of both sides.
- Write and solve two equations using both the positive and negative roots.

METHOD 5: USE THE QUADRATIC FORMULA

The **Quadratic Formula** can be used to solve *any* quadratic equation once it is in the form $ax^2 + bx + c = 0$.

> The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The *discriminant* is part of the **Quadratic Formula** that can be evaluated to determine whether the quadratic equation has 2, 1, or 0 solutions.
- The **discriminant** is $b^2 - 4ac$.
  - If the value of the discriminant is positive, the equation has 2 real solutions.
  - If the value of the discriminant is 0, the equation has 1 real solution.
  - If the value of the discriminant is negative, the equation has no real solutions.

Sincerely,

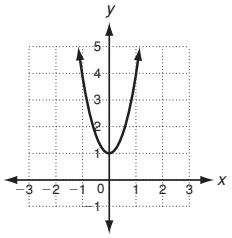
Name _____ Date _____ Class _____

CHAPTER 9

At-Home Practice

Solving Quadratic Equations

1. Match the graphed quadratic function to its related equation.



A. $x^2 + 3 = 0$ **C.** $3x^2 = 1$

B. $3x^2 + 1 = 0$ **D.** $x^2 + 3x = 0$

Use the Zero Product Property to solve each equation.

2. $(x - 5)(x - 9) = 0$ **3.** $x(x + 13) = 0$ **4.** $x(x - 1) = 0$

_____ _____ _____

Solve each equation by factoring.

5. $x^2 - 9x + 20 = 0$ **6.** $x^2 - 2x = 8$ **7.** $x^2 + 3x = 18$ **8.** $x^2 + 4x = 32$

_____ _____ _____ _____

Solve each equation by using square roots.

9. $51 = x^2 + 26$ **10.** $7x^2 = 343$ **11.** $2x^2 + 5 = 167$

_____ _____ _____

Complete the square for each expression.

12. $x^2 + 8x + \square$ **13.** $x^2 - 10x + \square$ **14.** $x^2 - 6x + \square$

_____ _____ _____

Use the discriminant to find the number of solutions of each quadratic equation.

15. $6x^2 - 3x = 5$ **16.** $x^2 + x + 1 = 0$ **17.** $x^2 + 18x + 81 = 0$

_____ _____ _____

Solve using the Quadratic Formula. Round your answers to the nearest hundredth.

18. $x^2 - 6x = -8$ **19.** $2x^2 + 4x = 7$ **20.** $10x^2 - 3x = 1$

_____ _____ _____

Answers: 1. B; **2.** 5, 9; **3.** 0, −13; **4.** 0, 1; **5.** 5, 4; **6.** 4, −2; **7.** −6, 3; **8.** −8, 4; **9.** 5, −5; **10.** 7, −7; **11.** 9, −9; **12.** 16; **13.** 25; **14.** 9; **15.** 2; **16.** 0; **17.** 1; **18.** 2, 4; **19.** −3.12, 1.12; **20.** 0.20, 0.50

Name _____ Date _____ Class _____

CHAPTER 9

Family Fun

Lose Your Chips

Objective: To practice finding the discriminant of a quadratic equation.

Materials: deck of cards
10 chips per player (or pennies)

Directions:

- Number cards are worth face value, Ace $= 1$, King $= \frac{3}{4}$, Queen $= \frac{3}{4}$, and Jack $= \frac{1}{4}$
- The player with the highest number goes first. If tied, the order from highest to lowest is spades, clubs, hearts, and diamonds.
- The player draws 3 cards and uses these cards to write a quadratic equation. Black cards are positive values, red cards are negative values.

  The first card is the *a*-value for the quadratic equation.

  The second card is the *b*-value.

  The third card is the *c*-value.
- The player is to then find the number of solutions to his or her quadratic equation by finding the discriminant without using a calculator.

Scoring:

- If the discriminant is positive, the player turns in 2 chips.
- If the discriminant is 0, the player turns in 1 chip.
- If the discriminant is negative, the player cannot turn in any chips.
- Another player may challenge the calculation of the discriminant.
  - o If there's an error in the calculation, the challenging player gives 2 chips to the player whose calculation was incorrect (you want to get rid of chips!).
  - o If the calculation was correct, that player gives one chip to the player who challenged the calculation.
- The first player to get rid of all of his or her chips wins.