CHAPTER 8

# Family Letter

## Section A

**What We Are Learning**

### Factoring Methods

**VOCABULARY**
These are the math words we are learning:

**greatest common factor (GCF)** the greatest factor shared by two or more whole numbers

**prime factorization** A representation of a number as the product of its prime-number factors.

*Dear Family,*

In this section, students will be revisiting some familiar concepts from past courses and applying these ideas to factoring polynomials.

Students know that a *product* is the result of two or more factors multiplied together. Students will use this terminology to study *prime factorization*.

$$\begin{array}{ccccc} 2 & \times & 9 & = & 18 \\ \uparrow & & \uparrow & & \uparrow \\ \text{factor} & \times & \text{factor} & = & \text{product} \end{array}$$

The **prime factorization** of a number is when the factors of the number are all prime numbers:

$$2 \times 3 \times 3 = 18$$

A *prime number* is a whole number that is not evenly divisible by any other whole number except for 1. The prime numbers between 1 and 10 are 2, 3, 5, and 7.

Students will learn two methods to write the prime factorization of a number. Have the student describe these methods to you:

1. Factor tree
2. Ladder Method

Students will use prime factorization when finding the **greatest common factor**, or GCF, of two or more terms. The **GCF** is the greatest factor that two or more terms have in common. Students will learn two methods for finding the GCF.

**Find the GCF of 8 and 20.**

**1.** List the factors:

$$\begin{array}{rl} 8: & 1, 2, 4, 8 \\ 20: & 1, 2, 4, 5, 10, 20 \end{array}$$

The greatest factor 8 and 20 have in common is 4

**2.** Use prime factorization:

$$\begin{array}{rcl} 8 & = & 2 \cdot 2 \cdot 2 \\ 20 & = & 2 \cdot 2 \cdot 5 \\ & & \downarrow \quad \downarrow \\ \text{GCF} & = & 2 \cdot 2 = 4 \end{array}$$

Circle the prime factors that the numbers have in common.

CHAPTER 8

# Family Letter

## *Section A* continued

In this section, students will use the GCF as a method for factoring polynomials.

**Factor $3x^2 + 6x$. Check your answer.**

**Ask:** *What factors do the terms* $3x^2$ *and* $6x$ *have in common*?

**Answer:** 3 and $x$

**Action:** Write the terms as products using the GCF as a factor:

$$\underbrace{3x}_{\text{GCF}}(x + 2) \qquad \text{Factored polynomial}$$

**Say:** Now, check the answer by multiplying the factors.

$$\begin{aligned} 3x(x + 2) &= 3x(x) + 3x(2) \\ &= 3x_2 + 6x \end{aligned}$$

Use the Distributive Property to distribute the $3x$ to each term.

The answer checks; the product is the original polynomial.

***Hot Tip!*** Checking answers helps to catch simple math errors.

Factoring methods for polynomials with three terms, *trinomials*, will be introduced to students once they master factoring with the GCF.

In the previous chapter, students multiplied binomials using the *FOIL* method to create trinomials. In this chapter, students will reverse that process. Given a trinomial, students will factor it into two binomials.

$x^2 + 3x + 2$ factors into $(x + 2)(x + 1)$.

The steps to factor trinomials are straightforward but require much practice for students to master. Encourage students to look at

- the operation signs before each term of a trinomial
- the coefficient of the $x$-term

There are numerous examples in the textbook that can be easily followed when working homework problems. Students should always take the time to use the FOIL method to verify that the trinomial is factored correctly.

*Sincerely,*

Name ______________________________ Date ___________ Class ____________

CHAPTER **8A**

# At-Home Practice

## *Factoring Methods*

**Write the prime factorization of each number.**

**1.** 85 ______________

**2.** 96 ______________

**3.** 240 ______________

**4.** 67 ______________

**Find the GCF of each pair of monomials.**

**5.** $-8s^2$ and $24s^4$ ______________

**6.** $-5z^3$ and $40z^5$ ______________

**7.** $-16d^2$ and 40 ______________

**8.** Which set of numbers has a GCF greater than 12? ______________________

**A.** 6 and 12 **B.** 12 and 18 **C.** 36 and 54 **D.** 40 and 65

**9.** The area of a rectangle is represented by $6x^2 - 14x - 12$. Which of the following could represent the area as a product of the length and width of the rectangle? ______

**F.** $(3x - 4)(2x + 3)$ **G.** $(6x + 4)(x - 3)$ **H.** $(6x - 2)(x + 6)$ **J.** $(6x - 3)(x + 4)$

**Factor each polynomial. Check your answer.**

**10.** $7d + 56d^3$ ______________

**11.** $10g^3 - 4g^2$ ______________

**12.** $32f + 8f^2 - 16f^4$ ______________

**13.** $6a^5b - 9ab^2$ ______________

**Factor each polynomial by grouping. Check your answer.**

**14.** $x^2 + 3x + 4x + 12$ ______________

**15.** $12x^2 - 15x + 10 - 8x$ ______________

**16.** $x^2 + 2x - 4x - 8$ ______________

**Factor each trinomial. Check your answer.**

**17.** $x^2 - 3x - 10$ ______________

**18.** $x^2 - 9x + 18$ ______________

**19.** $x^2 + x - 20$ ______________

**20.** $4x^2 + 15x + 9$ ______________

**21.** $10x^2 - x - 3$ ______________

**22.** $12x^2 - 7x - 12$ ______________

**Answers: 1.** $5 \cdot 17$; **2.** $2^5 \cdot 3$; **3.** $2^4 \cdot 3 \cdot 5$; **4.** $1 \cdot 67$; **5.** $8s^2$; **6.** $5z^3$; **7.** 8; **8.** C; **9.** G; **10.** $7d(1 + 8d^2)$; **11.** $2g^2(5g - 2)$; **12.** $8f(4 + f - 2f^3)$; **13.** $3ab(2a^4 - 3b)$; **14.** $(x + 4)(x + 3)$; **15.** $(3x - 2)(4x - 5)$; **16.** $(x + 2)(x - 4)$; **17.** $(x - 5)(x + 2)$; **18.** $(x - 3)(x - 6)$; **19.** $(x + 5)(x - 4)$; **20.** $(4x + 3)(x + 3)$; **21.** $(5x - 3)(2x + 1)$; **22.** $(3x - 4)(4x + 3)$

Name ______________________________________ Date ___________ Class ____________

CHAPTER 8A

# Family Fun

## *Factoring Finish*

**Objective:** To practice various methods of factoring.

**Materials:** number cube
game marker for each player

**Directions:**

- Each player rolls the number cube. The player who rolls the largest prime number goes first.
- A player rolls the number cube and moves his or her marker the indicated number of spaces on the game board. If a math problem on the square has already been completed, move back until you reach a math problem that has not been done (or Start).
- The player must then follow the instruction on the space. Math answers must be shown before the next player rolls. At that point, any player can challenge the answer.
- If the challenging player is correct, he or she may advance his or her marker two spaces forward and the player who originally answered the problem wrong must move his or her marker back to START. If the challenging player is wrong, he or she must move his or her marker back to START.
- The player who reaches FINISH first is the winner.

| | | | | |
|---|---|---|---|---|
| **START** | Find the GCF of $10x^4$ and $35x$. | Factor completely: $x^2 - 4x + 3$ | Roll Again | Write the prime factorization of 436. |
| | | | | Factor completely: $9x^2 - 6x + 1$ |
| Write the prime factorization of 48. | True or False: 4x is a factor of $18x + 28x^2$ | Switch places with another player. | Factor completely: $6x^3 - 42x$ | Factor completely: $16x^2 - 1$ |
| A rectangle has an area of $3x^2 + 20x + 25$. Find the length and width of the rectangle. | | | | |
| Find the GCF of $84x^3 - 16x^2 + 36x - 108x^4$. | Go back 5 spaces. | Factor completely: $4x^2 + 7xy + 3y^2$ | Write the prime factorization of 555. | **FINISH** |

CHAPTER 8

# Family Letter

## *Section B*

**What We Are Learning**

**Applying Factoring Methods**

*Dear Family,*

In Chapter 7, students were taught the patterns of two special products of binomials: difference of two squares and perfect squares. In this section, students will use these concepts to help identify and factor these products.

**PERFECT-SQUARE TRINOMIALS**

A perfect-square trinomial must meet the following conditions:

- The first and last terms must be perfect squares.
- The middle term is twice the product of the square roots of the first and last terms.

**Determine whether $4x^2 + 12x + 9$ is a perfect-square trinomial.**

**Ask:** Are the first and last terms perfect squares?

**Answer:** Yes: $4x^2$: $\sqrt{4x^2} = \sqrt{2 \cdot 2 \cdot x \cdot x} = \mathbf{2x}$

$9$: $\sqrt{9} = \sqrt{3 \cdot 3} = \mathbf{3}$

**Ask:** Is the middle term twice the product of the square roots of the first and last term?

**Answer:** Yes: $2(\mathbf{2x} \cdot \mathbf{3}) = 2(6x) = 12x$. So, $4x^2 + 12x + 9$ is a perfect-square trinomial.

When students can identify perfect-square trinomials, they will be able to quickly factor them mentally by following these patterns:

**For all numbers *a* and *b*:**

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

***Hot Tip!*** To factor a perfect-square trinomial, put the square root of the first term and the square root of the second term in a set of parentheses and place the operation sign between them.

**Factor $4x^2 + 12x + 9$.**

$$4x^2 + 12x + 9 = (2x\ \_\ 3)^2$$

$$= (2x + 3)^2$$

In Chapter 7, students learned that the **difference of two squares** has the form of $a^2 - b^2$.

**DIFFERENCES OF TWO SQUARES**

A polynomial is a difference of two squares if the following conditions are met:

- There are two terms and both are perfect squares.
- One term is subtracted from the other term.

**Determine whether $9x^2 - 25$ is a difference of two squares.**

**Ask:** Are the two terms perfect squares?

**Answer:** Yes: $\sqrt{9x^2} = 3x$ and $\sqrt{25} = 5$

**Ask:** Is one term being subtracted from the other term?

**Answer:** Yes

Since both answers are yes, then $9x^2 - 25$ is an example of a difference of two squares.

When students can identify a polynomial as a difference of two squares, they will be able to factor it quickly by following this rule:

**For all numbers *a* and *b*:**

$$a^2 - b^2 = (a + b)(a - b)$$

**Factor $9x^2 - 25$.**

Following the rule above, $9x^2 - 25 = (3x + 5)(3x - 5)$.

The student may need to factor out a quantity before using one of these methods to factor completely. It may help to have the student practice multiplying factors, then reverse the process to factor.

$3(3x + 5)(3x - 5) = 3(9x^2 - 25) = 27x^2 - 75$

$27x^2 - 75 = 3(9x^2 - 25) = 3(3x + 5)(3x - 5)$

*Sincerely,*

Name ______________________________ Date ____________ Class ____________

CHAPTER **8B**

# At-Home Practice

## *Applying Factoring Methods*

**Determine whether each trinomial is a perfect square. If so, factor. If not, explain.**

**1.** $x^2 + 4x + 4$ ______________________

**2.** $9x^2 - 30x + 25$ ______________________

**3.** $x^2 + 6x + 36$ ______________________

**4.** $16x^2 + 24x + 9$ ______________________

**Determine whether each trinomial is a difference of two squares. If so, factor. If not, explain.**

**5.** $x^2 - 49$ ______________________

**6.** $3x^2 - 81$ ______________________

**7.** $25x^2 - 121$ ______________________

**8.** $4x^2 - y^2$ ______________________

**Tell whether each expression is completely factored. If not, factor.**

**9.** $3(x^2 + 2x - 1)$ ____________

**10.** $6(x^2 + 5x - 6)$ ____________

**11.** $(5x - 15)(x + 3)$ ____________

**Factor each polynomial completely. Check your answer.**

**12.** $4x^2 + 2y - 6$ ____________

**13.** $3x^2 - 12$ ____________

**14.** $5x^2 + 20y^2$ ____________

**15.** $32x^2 + 48x + 16$ ____________

**16.** $49x^2 - 4$ ____________

**17.** $18x^6 + 60x^5 + 48x^4$ ____________

**Answers: 1.** yes, $(x + 2)^2$; **2.** yes, $(3x - 5)^2$; **3.** no, $6x \neq 2(x)(6)$; **4.** yes, $(4x + 3)^2$; **5.** yes, $(x - 7)(x + 7)$; **6.** no, 3 is not a perfect square; **7.** yes, $(5x - 11)(5x + 11)$; **8.** yes, $(2x - y)(2x + y)$; **9.** yes; **10.** no, $6(x + 6)(x - 1)$; **11.** no, $5(x - 3)(x + 3)$; **12.** $2(2x^2 + y - 3)$; **13.** $3(x - 2)(x + 2)$; **14.** $5(x^2 + 4y^2)$; **15.** $16(2x + 1)(x + 1)$; **16.** $(7x - 2)(7x + 2)$; **17.** $6x^4(3x + 4)(x + 2)$

Name ______________________________________ Date ___________ Class ____________

CHAPTER
8B

# Family Fun

## *Make It Perfect*

**Objective:** To create and factor perfect-square trinomials and difference of two square binomials.

**Materials:** polynomial slips (below)
paper bag or hat

**Directions:**

- Cut out the slips below and place them in the bag.
- Pass the bag to each player. Each player randomly selects one slip without looking at it.
- Players look at their slips at the same time.
- A player may change *one* term in the polynomial to make it a perfect-square trinomial or a difference of two squares binomial
- The first player to change his or her polynomial into a perfect-square trinomial or a difference of two squares binomial *and* factor it gets 1 point.
- Any player may challenge the winning factorization. If the factorization is incorrect, the challenger gets 2 points, the player with the incorrect factorization gets $-1$.
- The first player to get 5 points wins the game

| | | | | |
|---|---|---|---|---|
| $x^2 + 8x + 64$ | $6x^2 - 25$ | $4x^2 + 4x + 4$ | $49x^2 - 10$ | $x^2 - 30x - 45$ |
| $9x^2 - 18$ | $16x^2 - 12x + 4$ | $3x^2 - 64$ | $5x^2 - 2x + 1$ | $40x^2 - 144$ |
| $4x^2 + 28x - 36$ | $x^2 - 2$ | $9x^2 + 60x + 50$ | $4x^2 - 24$ | $25x^2 - 7x + 4$ |