CHAPTER 6

Family Letter
Section A

What We Are Learning

Systems of Linear Equations

VOCABULARY
These are the math words we are learning:

consistent system a system with at least one solution

dependent system a system that has an infinite number of solutions

inconsistent system a system that has no solutions

independent system a system with exactly one solution

system of linear equations a set of two or more linear equations containing two or more variables

Dear Family,

In this section, the building blocks of algebra will become apparent as students study systems of linear equations. Students apply their graphing and equation-solving techniques to determine the solution(s) of a system.

A **system of linear equations** is a set of at least two linear equations.

A system of 2 linear equations: $\begin{cases} y = x + 2 \\ y = 2x + 3 \end{cases}$

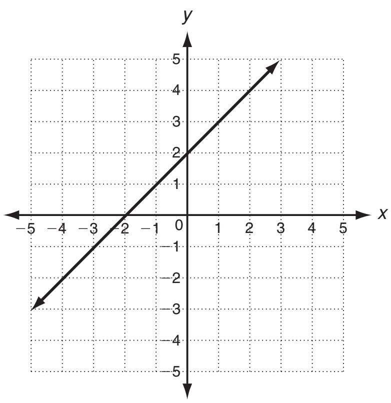
To solve a system of linear equations, students will learn three different methods. Each lesson in this section is dedicated to one solving method.

- Graphing
- Substitution
- Elimination

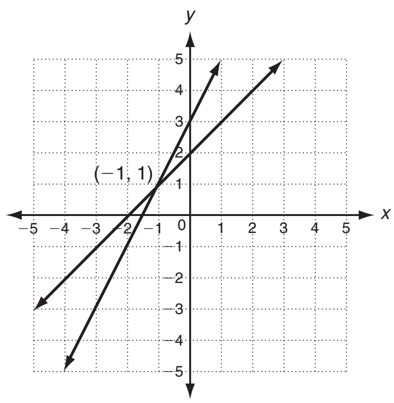
Students are taught different solving methods because often one method may be easier and faster to use than the others.

Solve the system: $\begin{cases} y = x + 2 \\ y = 2x + 3 \end{cases}$ **by graphing.**

The student will graph the first equation: $y = x + 2$.



Then the student will graph the second equation $y = 2x + 3$ *on the same grid.*



The point (−1, 1) is the intersection of the two lines, thus, the solution to the system is (−1, 1), which means $x = -1$ and $y = 1$.

The student should check this solution in both equations:

$\begin{cases} y = x + 2 \\ y = 2x + 3 \end{cases} \longrightarrow \begin{matrix} \mathbf{1} = \mathbf{-1} + 2 \\ \mathbf{1} = 2(\mathbf{-1}) + 3 \end{matrix} \longrightarrow \begin{matrix} 1 = 1✓ \\ 1 = 1✓ \end{matrix}$

Solve the system: $\begin{cases} y = x + 2 \\ y = 2x + 3 \end{cases}$ **by substitution.**

When an equation has been solved for a variable, such as $y = x + 2$, the left side and right side are equivalent. This means that you can substitute $x + 2$ for y in the second equation:

$$\begin{cases} y = x + 2 \\ \mathbf{y} = 2x + 3 \end{cases} \text{ substitute } \longrightarrow \begin{cases} y = x + 2 \\ \mathbf{x + 2} = 2x + 3 \end{cases}$$

Now the second equation is easy to solve for x.

$$\begin{aligned} x + 2 &= 2x + 3 \\ -1 + x &= 2x \\ -1 &= x \end{aligned}$$

Now substitute $x = -1$ into the first equation, $y = x + 2$, and solve for y. The solution is $(-1, 1)$ as before. Again, checking is important.

Solve the system: $\begin{cases} y = x + 2 \\ y = 2x + 3 \end{cases}$ **by elimination.**

The method of elimination is used to eliminate one of the two variables in the equations by adding or subtracting multiples of the equations.

The student might notice that the first equation can be subtracted from the second and the y's will be eliminated.

$$\begin{aligned} y &= 2x + 3 \\ y &= x + 2 \\ \hline 0 &= x + 1 \end{aligned}$$

$\longleftarrow$ $x = -1$ as above, substitute -1 for x in the first equation to find $y = 1$ as above.

The student should know that a system of equations may have *no* solutions or an *infinite number* of solutions. Have the student illustrate how these situations are possible.

Sincerely,

Name ______________________ Date __________ Class __________

CHAPTER 6

At-Home Practice

Systems of Linear Equations

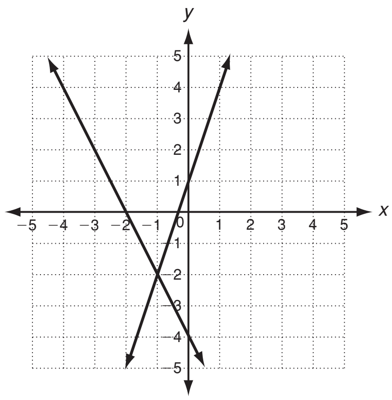
Tell whether the ordered pair is a solution of the given system.

1. $(-1, 12)$; $\begin{cases} y = -4x + 8 \\ y = -3x + 9 \end{cases}$

2. $(0, -1)$; $\begin{cases} y = 4x - 1 \\ y = -3x + 6 \end{cases}$

3. $(2, 2)$; $\begin{cases} y = \frac{x}{2} + 1 \\ y + 4x = 10 \end{cases}$

4. The graph represents which system of equations?



A. $\begin{cases} y = 2x + 1 \\ y = -x - 4 \end{cases}$

B. $\begin{cases} y = 3x + 1 \\ y = -2x - 4 \end{cases}$

C. $\begin{cases} y = 3x - 4 \\ y = 2x + 1 \end{cases}$

D. $\begin{cases} y = 2x + 1 \\ y = -3x - 4 \end{cases}$

Solve each system by substitution.

5. $\begin{cases} y = -2x + 10 \\ y = -4x + 22 \end{cases}$

6. $\begin{cases} 12x - 6y = -3 \\ y = 2 + 3x \end{cases}$

7. $\begin{cases} 2x + y = 4 \\ y = -4x - 10 \end{cases}$

Solve each system by elimination.

8. $\begin{cases} 3x + y = 2 \\ 2x - y = -12 \end{cases}$

9. $\begin{cases} 2x + 2y = 4 \\ 4x + 2y = -2 \end{cases}$

10. $\begin{cases} x + 2y = -3 \\ 3x + 2y = 15 \end{cases}$

Solve each system of linear equations.

11. $\begin{cases} 2x + y = 8 \\ y = -2x + 7 \end{cases}$

12. $\begin{cases} 2y = x - 6 \\ -4x + 8y = -2 \end{cases}$

13. $\begin{cases} y = -6x + 8 \\ 12x + 2y = 16 \end{cases}$

Answers: 1. yes; **2.** no; **3.** yes; **4.** B; **5.** $(6, -2)$; **6.** $\left(-\frac{3}{2}, -\frac{5}{2}\right)$; **7.** $(-7, 18)$; **8.** $(-2, 8)$; **9.** $(-3, 5)$; **10.** $(9, -6)$; **11.** no solution; **12.** no solution; **13.** infinite solutions

Name ______________________________________ Date ___________ Class ____________

CHAPTER 6

Family Fun

Pick and Roll

Objective: To create and solve systems of equations using substitution, graphing, or elimination.

Materials: deck of regular playing cards

Black cards are *positive* values; red cards are *negative* values.

Number cards are used at face value, king $= \frac{1}{2}$, queen $= \frac{2}{3}$, jack $= \frac{3}{4}$, ace $= 1$

Directions:

- Shuffle the deck and place it face down.
- Player 1 selects two cards from Stack A and writes the first equation of the system. The first card selected is the slope, the second is the *y*-intercept:

  Example: Player selects 5 of hearts and king of clubs:

  The first equation is $y = -5x + \frac{1}{2}$
- Player 1 then selects two more cards and writes the second equation of the system.
- Player 1 selects 1 more card.

If the 5th card is a(n):	the player will solve the system by:
Jack, Queen, or King	substitution
Ace	graphing
2, 3, 4, 5, 6, 7, 8, 9, or 10	elimination

- Player 2 is to check Player 1's solution.
- The solving player gets 1 point if the system checks. The checking player gets 2 points if the solution does not check.
- Play again, with Player 2 selecting the cards from the deck and writing and solving the system.
- Play until one player gets 5 points.

Family Letter
Section B

What We Are Learning

Linear Inequalities

VOCABULARY
These are the math words we are learning:

linear inequality similar to a linear equation, but the equal sign is replaced with an inequality sign

solution of a linear inequality any ordered pair that makes the inequality true

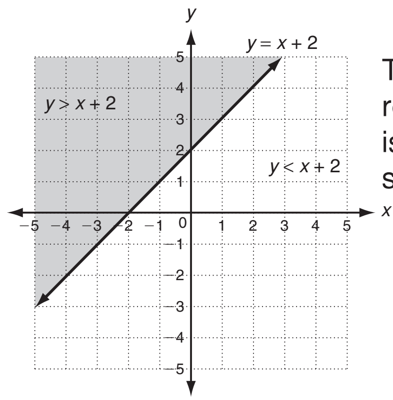
Dear Family,

Students have been practicing finding solutions for linear equations and systems of linear equations. In this section, students will use these skills to find the **solutions of a linear inequality** and the solutions to a **system of linear inequalities**.

Students are aware that the solutions of a linear equation can be graphed as a straight line and the solutions are all of the points on that line. The **solution of a linear inequality**, however, is a *region* on a coordinate plane; any point in the region is a solution of the inequality.

This is a graph of the solutions of the linear inequality $y \geq x + 2$.

The region $y > x + 2$ is shaded.



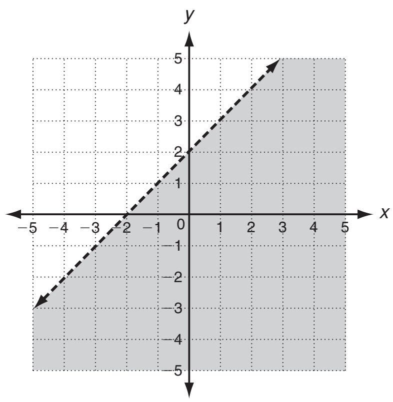
The solid line representing $y = x + 2$ is also part of the solution.

All of the ordered pairs *on or above* the line make the inequality true.

To Graph the Solutions of an Inequality

1. Replace the inequality symbol with an equal sign and solve the resulting linear equation.
2. If the symbol in the inequality is $<$ or $>$, the line should be dashed. Points on the line are *not* solutions of the inequality. If the inequality symbol is $\leq$ or $\geq$, the line should be solid. Points on the line *are* solutions of the inequality.
3. To determine the region to shade, choose a point not on the line and substitute it into the inequality. The origin, (0, 0), is the easiest point to use if it is not on the line.
4. If the selected point makes the inequality true, shade the region containing that given point. If the point does not make the inequality true, shade in the region that does not contain the point. In the inequality above, $y \geq x + 2$, (0, 0) is *not* a solution, so the other region is shaded.

Graph the solutions of $y - x < 2$**.**



1. Solve for y: $y < x + 2$.

2. Graph $y = x + 2$.

3. The line is dashed because the inequality symbol is $<$.

4. Substitute (0, 0) into the inequality.

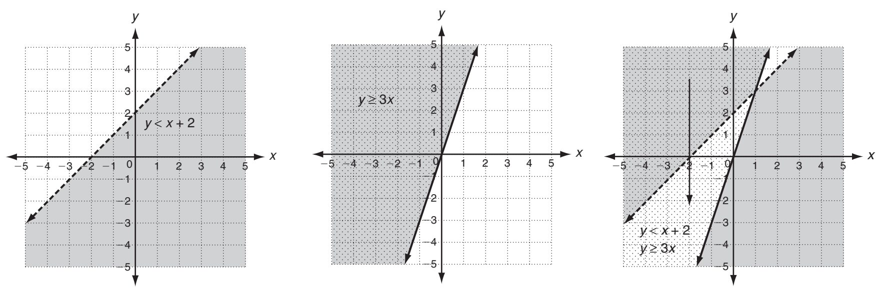
$y < x + 2$
$\mathbf{0} < \mathbf{0} + 2$
$0 < 2$ true

So shade the region that *includes* the point (0, 0).

Once students can effectively graph a linear inequality, they will move on to graphing **systems of linear inequalities**. The solutions of a system of inequalities are the **overlap** of the solutions of the inequalities that make up the system.

Graph the solutions of $\begin{cases} y < x + 2 \\ y \geq 3x \end{cases}$

The individual inequalities… The intersecting region is the solution set.



Always check to be sure an ordered pair is true for *both* inequalities.

It helps to have the student shade the first inequality in one color or pattern and the region for the other inequality in another color or pattern. This way, the overlap of the regions will be apparent.

Systems of equations and inequalities have many practical real-world applications. Ask your student to explain some of these applications and to point out other examples in daily life.

Sincerely,

Name ______________________ Date ____________ Class ____________

CHAPTER 6

At-Home Practice

Linear Inequalities

Tell whether the ordered pair is a solution of the inequality.

1. $(3, -4)$; $y < 2x + 4$ ____________

2. $(-2, -5)$; $-3x + 2y \leq -6$ ____________

3. $(0, -4)$; $-5y \leq -x + 4$ ____________

Match each linear inequality to the correct graph of its solution.

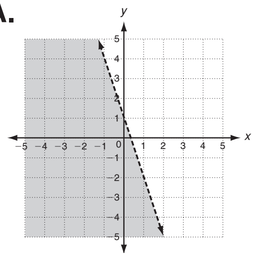
4. $y < -\frac{3}{4}x$ ____________

5. $y < -3x + 1$ ____________

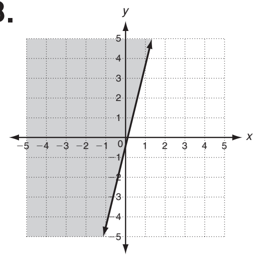
6. $y \geq 2x - 3$ ____________

7. $y \geq 4x - \frac{1}{2}$ ____________

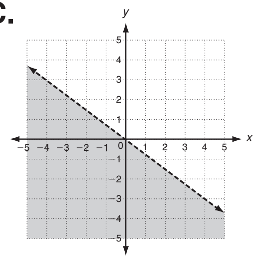
A.



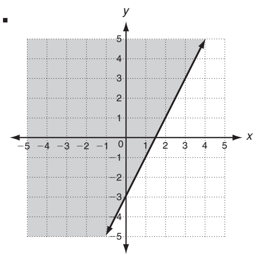
B.



C.



D.



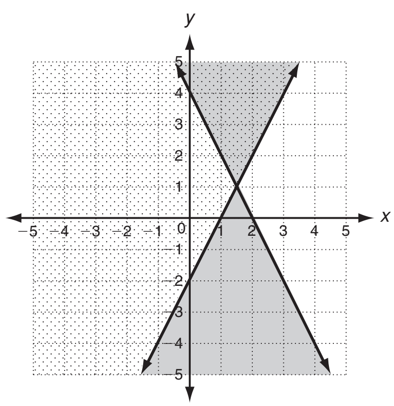
Tell whether the ordered pair is a solution of the given system.

8. $(4, 2)$; $\begin{cases} y < x + 4 \\ y > -2x + 3 \end{cases}$ ____________

9. $(-3, 5)$; $\begin{cases} y \leq 4x - 2 \\ y > 2x + 1 \end{cases}$ ____________

10. $(3, -1)$; $\begin{cases} 3x + 4y < 12 \\ -4x + 2y \leq 10 \end{cases}$ ____________

11. Which system of linear inequalities represents the given graph?



A. $\begin{cases} y \geq x + 2 \\ y \leq -2x - 4 \end{cases}$

C. $\begin{cases} 2y \leq 4x - 5 \\ -3y \geq x + 2 \end{cases}$

B. $\begin{cases} 4x + 3y \geq 6 \\ x - y \leq -2 \end{cases}$

D. $\begin{cases} y \leq -2x + 4 \\ y \geq 2x - 2 \end{cases}$

Answers: **1.** yes; **2.** no; **3.** no; **4.** C; **5.** A; **6.** D; **7.** B; **8.** yes; **9.** no; **10.** yes; **11.** D

Name ______________________________ Date __________ Class __________

CHAPTER 6

Family Fun

Find the Letter

Objective: To determine if an ordered pair is a solution to a linear inequality or a system of linear inequalities.

Materials: Game sheet and colored pencils

Directions:

For each square in the game sheet below:

- Determine whether the ordered pair is a solution of the given linear inequality or system of linear inequalities.
- If the ordered pair is a solution, shade the square blue.
- If the ordered pair is *not* a solution, shade the square red.

What letter of the alphabet is formed by the blue squares? _______

$(4, 6);\ y \le 4x + 2$	$(1, -1);\ \begin{cases} y > x \\ y < x - 3 \end{cases}$	$(-4, 3);\ 3x - 2y > -10$	$(4, 5);\ \begin{cases} y \ge 2x - 3 \\ y \le x + 3 \end{cases}$
$(1, -2);\ \begin{cases} \frac{2}{3}x + y < -5 \\ y \ge -\frac{1}{4}x \end{cases}$	$(3, -3);\ 2x + y < 16$	$(-2, -3);\ \begin{cases} y < -x \\ -4y > x + 8 \end{cases}$	$(5, -3);\ y \le -6x$
$(6, 3);\ 2y > x$	$(5, -3);\ \begin{cases} x > y + 5 \\ 2x + 5y < -3 \end{cases}$	$(-2, 3);\ 2x + y < 4$	$(-2, -1);\ \begin{cases} y < 3x \\ -y > -2x \end{cases}$
$(0, -6);\ \begin{cases} -2x - 3y > 2 \\ 2y < -x + 6 \end{cases}$	$\left(\frac{1}{4}, \frac{2}{3}\right);\ y \le 2x$	$(3, -4);\ \begin{cases} 3x - 4y < 20 \\ y > 3x \end{cases}$	$(-3, 6);\ 4y > -4x$

Answer: X