CHAPTER 5

Family Letter

Section A

What We Are Learning

Characteristics of Linear Functions

VOCABULARY
These are the math words we are learning:

constant of variation a nonzero constant, k used in a direct variation

direct variation a special type of linear relationship that can be written in the form $y = kx$, where k is a nonzero constant

linear equation the equation of a linear function

linear function a function whose graph forms a straight line

slope the ratio of rise to run for any two points on a line

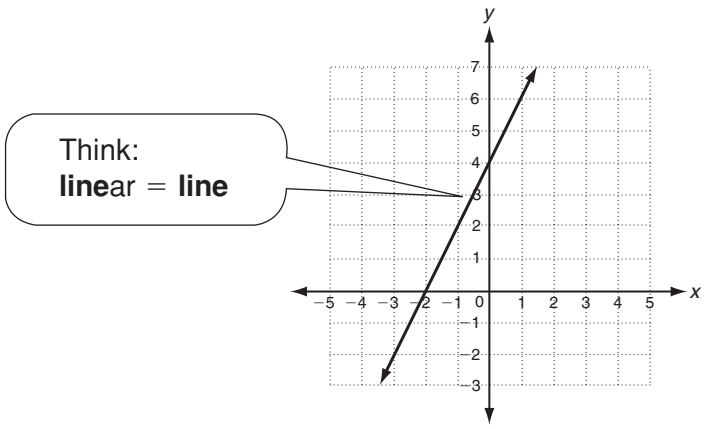
***y*-intercept** the y-coordinate of any point where a graph intersects the y-axis

***x*-intercept** the x-coordinate of any point where a graph intersects the x-axis

Dear Family,

In this section, students will expand their knowledge of functions and recognize and apply the traits of a specific type of function called a **linear function**.

A function is linear if its graph is a straight line, such as the one representing $y = 2x + 4$ below.



A linear function can also be described by a **linear equation**. When a function is written as an equation, its solutions are all the values that make the equation true.

You can see by the line that there are an infinite number of solutions. For $y = 2x + 4$, when $x = 0$, $y = 4$, so (0, 4) is one solution. When $x = -2$, $y = 0$, so $(-2, 0)$ is another solution.

The two points mentioned above are important because they give unique information about the linear function.

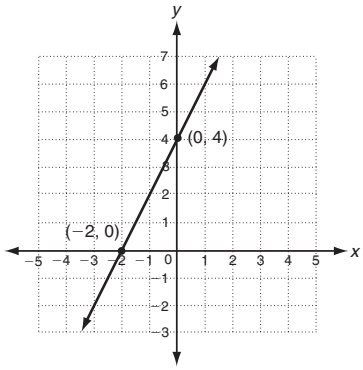
- The ***x*-intercept** is the point where the line crosses the x-axis. The y-coordinate of this point is *always* 0. The x-intercept, –2, occurs at (–2, 0).
- The ***y*-intercept** is the point where the line crosses the y-axis. The x-coordinate of this point is *always* 0. The y-intercept, 4, occurs at (0, 4).

For any two points, there is exactly one line that can be drawn that passes through both. So a student who can find the intercepts quickly can use these two points to graph the linear equation.

Graph the equation of the line $y = 2x + 4$.
Find the intercepts:

***x*-intercept**	***y*-intercept**
replace y with 0 and solve for x	replace x with 0 and solve for y
$y = 2x + 4$	$y = 2x + 4$
$\mathbf{0} = 2x + 4$	$y = 2(\mathbf{0}) + 4$
$-4 = 2x$	$y = 4$
$x = -2$	

The intercepts are found at $(-2, 0)$ and $(0, 4)$. Graphing these points results in the line shown.



The student will need to determine the tilt or **slope** of a line. Slope is a ratio showing the constant rate of increase or decrease of a linear function. It is represented by the letter, m.

To find the slope of a line, students can use two points on the line, such as the x- and y-intercepts, and the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← The difference in y-values / ← The difference in x-values.

Find the slope of the line that contains (0, 4) and (−2, 0).

Substitute (0, 4) for (x_1, y_1) and $(-2, 0)$ for (x_2, y_2) in the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-2 - 0} = \frac{-4}{-2} = 2$$

The slope is 2.

Use the graph of the line above and see that any time you move 1 unit to the right, you move 2 units up.

Understanding intercepts and slope is key to learning about linear functions.

Sincerely,

Name _____ Date _____ Class _____

CHAPTER 5

At-Home Practice

Characteristics of Linear Functions

Without graphing, tell whether each point is on the graph of the given line.

1. $y = 2x + 5$; (3, 11) _____

2. $y = -4x - 6$; (−2, 2) _____

Match each linear equation with its corresponding graph. Then determine if the graph represents a function.

3. $2x + y = 4$ _____

_____

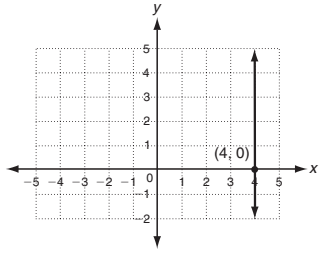
4. $x = 4$ _____

_____

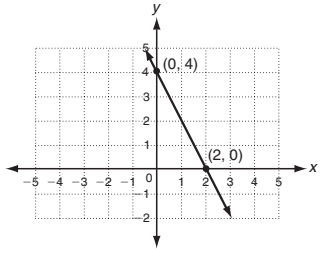
5. $2y = 2x + 8$ _____

_____

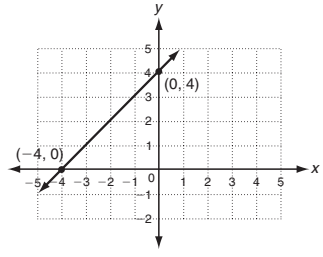
A.



B.



C.



Find the *x*- and *y*-intercepts.

6. $6x - 3y = 12$

_____

7. $-\frac{4}{5}x + \frac{1}{5}y = -20$

_____

8. $y = 4x - 3$

_____

Find the slope of the line described by each equation.

9. $y = 4x - 12$ _____

10. $y = -10x + 6$ _____

Find the slope of the line that passes through each pair of points.

11. (3, 6) and (5, 2)

_____

12. (−2, 4) and (5, 8)

_____

13. (4, 1) and (6, 1)

_____

14. (3, 4) and (4, 3)

_____

Answers: **1.** yes; **2.** yes; **3.** B, yes; **4.** A, no; **5.** C, yes; **6.** *x*-int: 2, *y*-int: −4; **7.** *x*-int: 25, *y*-int: −100; **8.** *x*-int: $\frac{3}{4}$, *y*-int: −3; **9.** 4; **10.** −10; **11.** $m = -2$; **12.** $m = \frac{4}{7}$; **13.** $m = 0$; **14.** $m = -1$

Name _____ Date _____ Class _____

CHAPTER 5

Family Fun

Rise and Fall

Objective: To find the slope of a line when given two points.

Materials: a coordinate grid, slips of paper, bowl or hat

Directions:

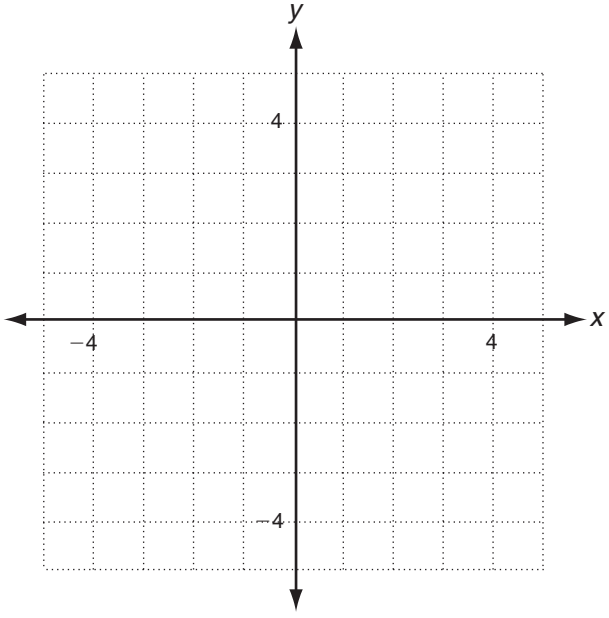
- Write each ordered pair shown below on separate slips of paper and place the papers into a bowl or hat.

(5, 3)	**(−1, 2)**	**(0, 4)**	**(−3, −2)**
(2, −2)	**(1, 3)**	**(−1, −3)**	**(2, −4)**
(4, 0)	**(−5, 3)**	**(2, 1)**	**(2, 2)**
(3, 4)	**(4, 2)**	**(3, 1)**	**(5, 1)**

- A player randomly selects a slip of paper from the hat, plots the point on the grid, and draws a line connecting the new point with the previously plotted point (use the point (0, 0) for the first play of the game).
- The player finds the slope of the line.
- Depending on the value of the slope, the player is awarded the following points:

  3 points for a positive slope **0 point for no slope**
  2 points for a negative slope **−1 point for undefined slope**

- The next player then randomly selects a slip of paper and repeats the process described above until a player accumulates **10 points**.



CHAPTER 5

Family Letter

Section B

What We Are Learning

Using Linear Functions

VOCABULARY
These are the math words we are learning:

family of functions functions whose graphs share the same basic characteristics, such as linear functions.

parallel lines lines with the same slopes that do not intersect

perpendicular lines lines that intersect at a right angle

Dear Family,

The student has been practicing finding the slope and the intercepts of a line from a linear function or a graph of a line. In this section, the student will:

- graph a line
- write an equation that describes a line

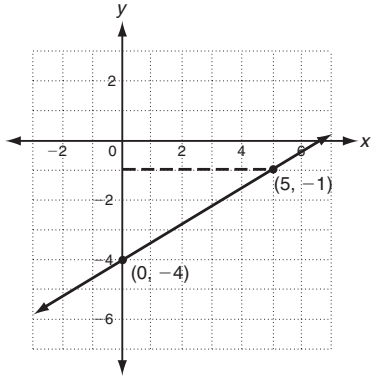
Your student now can use the *y*-intercept and the slope of a line to graph the line by using these steps.

Steps to Graph a Line Using *y*-intercept and Slope

Step 1 Plot the *y*-intercept point. If the *y*-intercept is -4, the point where the line crosses the *y*-axis is $(0,-4)$. Plot a point at $(0, -4)$.

Step 2 Use the slope to find and plot another point on the line. Slope is $\frac{\text{change in } y}{\text{change in } x}$. So for a slope of $\frac{3}{5}$ count 3 units up and 5 units right from $(0, 4)$ and plot the point $(5, -1)$.

Step 3 Connect the two points with a line.



The student can also use the slope along with the *y*-intercept or another point on the line to write the equation of that line. Students will study two different equivalent forms for writing a linear equation.

- The **slope-intercept form**: $y = mx + b$, where m represents the slope of the line and b represents the *y*-intercept. For example, $y = \mathbf{\frac{3}{5}}x - \mathbf{4}$
- The **point-slope form**: $y - y_1 = m(x - x_1)$, where m represents the slope of the line, and (x_1, y_1) represents a point on the line. For example, $y - (\mathbf{-1}) = \mathbf{\frac{3}{5}}(x - \mathbf{5})$

Family Letter

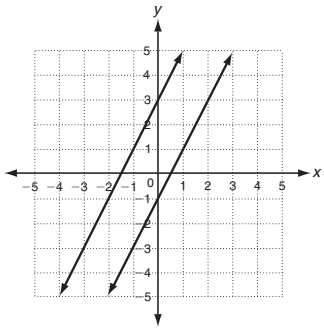
Section B continued

Once the student understands the basics of linear functions, he or she will apply these skills as well as equation-solving skills to write and graph linear equations given minimal information.

Also in this section, the student is asked to determine if the graphs of two linear equations are parallel or perpendicular lines.

Parallel lines are lines that are in the same plane that never intersect, like railroad tracks or parallel bars.

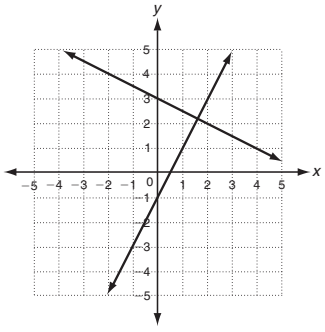
- Parallel lines have equal slopes.



Example The graphs of $y = \mathbf{2}x - 1$ and $y = \mathbf{2}x + 3$ are parallel lines because both are linear functions with a slope of 2.

Perpendicular lines are lines that intersect at right angles, such as two roads at a stop sign.

- The slopes of perpendicular lines are opposite reciprocals.



Example The graphs of $y = \mathbf{2}x - 1$ and $y = -\frac{\mathbf{1}}{\mathbf{2}}x + 3$ are *perpendicular lines* because they are linear functions with slopes of 2 and $-\frac{1}{2}$, which are opposite reciprocals. To check, the product of the slopes should be -1: $2\left(-\frac{1}{2}\right) = -1$ ✓

Applying and practicing these new skills will help the student become proficient in graphing, writing, and interpreting linear functions; a skill needed for solving systems of equations.

It is essential that the student completes all homework concerning these concepts, which are among the most important in algebra, as well as a good indicator of later success in testing and in future mathematics courses.

Sincerely,

Name _____ Date _____ Class _____

CHAPTER 5

At-Home Practice

Using Linear Functions

Write each equation in slope-intercept form.

1. $3x + y = 6$

_____

2. $6x - 8y = 24$

_____

3. $8x - 2y = 2$

_____

4. $-4x + 6y = 20$

_____

5. Which function has the same y-intercept as $2y = 4x - 12$? _____

A. $y = x + 2$ **B.** $y = x + 6$ **C.** $y = 2x - 6$ **D.** $y = x - 12$

Write an equation in point-slope form for the line with the given slope that contains the given point.

6. slope $= \frac{1}{4}$; $(4, 6)$

_____

7. slope $= -\frac{2}{3}$; $(2, 8)$

_____

Write an equation in slope-intercept form for the line with the given slope that contains the given point.

8. slope $= 6$; $(-3, -1)$

_____

9. slope $= 4$; $(0, 5)$

_____

Write an equation in slope-intercept form for the line that passes through the two points.

10. $(4, 2)$ and $(1, 0)$

_____

11. $(2, -1)$ and $(6, -3)$

_____

12. $(6, 1)$ and $(-8, 0)$

_____

13. $(3, -6)$ and $(2, 5)$

_____

14. A line contains $(1, 3)$ and $(5, 9)$. What is the slope? _____

What is the y-intercept? _____

Determine if each pair of lines is parallel or perpendicular.

15. $y = 2x - 6$
$y = 2x + 5$

_____

16. $y = \frac{1}{3}x - 8$
$y = -3x + 4$

_____

17. $4x - 2y = 6$
$y = -\frac{1}{2}x + 9$

_____

18. $\frac{1}{6}y = -\frac{1}{3}x$
$y = -2x$

_____

Answers: **1.** $y = -3x + 6$; **2.** $y = \frac{3}{4}x - 3$; **3.** $y = 4x - 1$; **4.** $y = \frac{2}{3}x + \frac{10}{3}$; **5.** C; **6.** $(y - 6) = \frac{1}{4}(x - 4)$; **7.** $(y - 8) = -\frac{2}{3}(x - 2)$; **8.** $y = 6x + 17$; **9.** $y = 4x + 5$; **10.** $y = \frac{2}{3}x - \frac{2}{3}$; **11.** $y = -\frac{1}{2}x$; **12.** $y = \frac{1}{14}x + \frac{4}{7}$; **13.** $y = -11x + 27$; **14.** $\frac{3}{2}$; $\frac{3}{2}$; **15.** parallel; **16.** perpendicular; **17.** perpendicular; **18.** parallel

Name _____ Date _____ Class _____

CHAPTER 5

Family Fun

Find the Line

Objective: To write equations of lines and to identify perpendicular and/or parallel lines.

Materials: Game sheet
Game slips
Paper bag (or hat or bowl)

Directions:

- Cut out each ordered pair game slip below.
- Place the slips of papers into the bag.
- Each player randomly selects two slips of paper from the bag and completes the following:

1. Write the equation of the line that passes through the two points selected in slope-intercept form.

2. Record the ordered pairs and the equation of the line on the game sheet.

3. Replace the slips of paper back into the bag after each turn.

4. The first person to write an equation of a line that is parallel or perpendicular to a line previously written is the winner of that round.

(4, 2)	(6, 1)	(2, 4)	(5, −1)	(1, 2)
(3, 4)	(4, 6)	(5, 3)	(6, 1)	(0, 0)
(1, 4)	(−4, −1)	(3, 3)	(−5, 1)	(3, −3)

Game Sheet

Round	Ordered Pairs	Equation of the Line
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		