CHAPTER 11

Family Letter

Section A

What We Are Learning

Radical Functions and Equations

VOCABULARY
These are the math words we are learning:

like radicals
square root expressions with the same radicand

radical equation
an equation that contains a variable under a radical

radical expression
an expression that contains a radical symbol, $\sqrt{}$

radicand
the expression under the radical symbol

square-root function
a function whose rule contains a variable under a square-root sign

Dear Family,

In this section, students will primarily apply mathematical operations to radical expressions, specifically **square-root functions**. A **radical expression** is any expression that contains a radical sign, $\sqrt{}$, which indicates the nonnegative square root.

Examples of Radical Expressions:

$\sqrt{2}$

$\sqrt{2x + 5}$

$5\sqrt{4x} + 7$

> A **radicand** is the expression under the radical sign. It may contain numbers, variables, or both.

As the students work with square-root functions, they will need to remember a few key concepts.

- The square-root of a negative number is not a real number. So the domain (possible x-values) of a square-root function is restricted to numbers that make the value under the radical sign greater than or equal to 0.

Find the domain of $\sqrt{x - 2} + 7$ **.**

Solve the inequality $x - 2 \geq 0$. The domain is the set of all numbers greater than or equal to 2.

To completely *simplify* a square-root expression:

- the radicand cannot contain a perfect square or be written as a fraction.
- the denominator cannot contain any square roots.

The following are not simplified:

~~$\sqrt{16}$~~ 16 is a perfect square.

~~$\sqrt{\frac{3}{5}}$~~ The radicand contains a fraction.

~~$\frac{3}{\sqrt{2}}$~~ The denominator contains a square-root.

Students will practice simplifying radical expressions as they add, subtract, multiply, and divide radical expressions. When adding or subtracting radical expressions, students will need to add **like radicals**, square-root expressions with the same radicand. This is just like adding and subtracting algebraic expressions with like terms.

Add $3\sqrt{5} + 6\sqrt{5}$.

$$3\sqrt{5} + 6\sqrt{5} = (3 + 6)\sqrt{5} = 9\sqrt{5}.$$

like radicals

Helpful Hint: Simplify all radicals in an expression before trying to identify like radicals.

Students will use various mathematical properties to multiply radial expressions, such as:

• Product Property of Square Roots

• Quotient Property of Square Roots

Ask your student to explain these properties. The textbook offers many examples that fully explain and illustrate how to multiply and divide radical expressions.

A radical expression is not considered simplified if there is a radical in the denominator of a fraction. A challenging concept for many students is how to *rationalize the denominator* to arrive at an equivalent, yet simplified, expression. Students will need to multiply the expression by a form of 1 to get a perfect-square radicand in the denominator.

Simplify $\frac{3}{\sqrt{2}}$.

$$\frac{3}{\sqrt{2}} = \frac{\sqrt{\mathbf{2}}}{\sqrt{\mathbf{2}}} \cdot \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}} = \frac{3\sqrt{2}}{2}$$

Having a strong foundation on the basics of radical expressions will help students when learning about more complex radical expressions, cube roots, logarithms, and trigonometry.

Sincerely,

Name ______________________ Date ____________ Class ____________

CHAPTER 11 At-Home Practice
Radical Functions and Equations

Find the domain of each square-root function.

1. $y = \sqrt{5x} + 2$ ____________

2. $y = \sqrt{x + 6}$ ____________

3. $y = \sqrt{3x - 9}$ ____________

Simplify. All variables represent nonnegative numbers.

4. $\sqrt{98}$ ____________

5. $\sqrt{192}$ ____________

6. $\sqrt{x^5y^4z^2}$ ____________

7. $\sqrt{\frac{225}{121}}$ ____________

8. $3\sqrt{5} + 4\sqrt{5}$ ____________

9. $\sqrt{12} + 4\sqrt{3}$ ____________

10. $6\sqrt{27} - 9\sqrt{3}$ ____________

11. $\sqrt{40} - \sqrt{49}$ ____________

Multiply. Write each product in simplest form. All variables represent nonnegative numbers.

12. $\sqrt{3}\,\sqrt{6}$ ____________

13. $(5 - \sqrt{7})\,(4 + \sqrt{7})$ ____________

14. $2\sqrt{3x}\,\sqrt{8x}$ ____________

Simplify each quotient.

15. $\frac{\sqrt{14}}{\sqrt{2}}$ ____________

16. $\frac{\sqrt{56}}{\sqrt{3}}$ ____________

17. $\frac{\sqrt{45}}{\sqrt{2}}$ ____________

Solve. Check your answer.

18. $\sqrt{3x + 1} = 8$ ____________

19. $2\sqrt{x} = 10$ ____________

20. $\sqrt{x - 3} = 3$ ____________

21. $\sqrt{2x} - 5 = 15$ ____________

Answers: 1. $x \geq 0$; **2.** $x \geq -6$; **3.** $x \geq 3$; **4.** $7\sqrt{2}$; **5.** $8\sqrt{3}$; **6.** $x^2y^2z\sqrt{x}$; **7.** $\frac{15}{11}$; **8.** $7\sqrt{5}$; **9.** $6\sqrt{3}$; **10.** $9\sqrt{3}$; **11.** $2\sqrt{10} - 7$; **12.** $3\sqrt{2}$; **13.** $13 + \sqrt{7}$; **14.** $4x\sqrt{6}$; **15.** $\sqrt{7}$; **16.** $\frac{2\sqrt{42}}{3}$; **17.** $\frac{3\sqrt{10}}{2}$; **18.** $x = 21$; **19.** $x = 25$; **20.** $x = 12$; **21.** $x = 200$

Name ______________________________ Date __________ Class __________

CHAPTER 11

Family Fun

A Radical Puzzle

Objective: To practice solving radical equations.

Materials: pencil

Directions:

Complete the puzzle by inserting simplified radicals to make each number sentence true.

$3\sqrt{2}$	$+$		$=$	$5\sqrt{2}$				$\sqrt{20}$
$-$				$+$				$\times$
	$-$	$\sqrt{2}$	$=$					
$=$		$\times$		$=$		$-$		$=$
				20	$\div$		$=$	$4\sqrt{5}$
		$+$				$=$		
		$3\sqrt{5}$		$12\sqrt{3}$	$\div$		$=$	
		$=$				$+$		
$4\sqrt{6}$	$\times$		$=$			$6\sqrt{5}$		

Answers:

$3\sqrt{2}$	$+$	$2\sqrt{2}$	$=$	$5\sqrt{2}$				$\sqrt{20}$
$-$				$+$				$\times$
$3\sqrt{2}$	$-$	$\sqrt{2}$	$=$	$2\sqrt{2}$		$10\sqrt{5}$		2
$=$		$\times$		$=$		$-$		$=$
0		$\sqrt{10}$		20	$\div$	$\sqrt{5}$	$=$	$4\sqrt{5}$
		$+$				$=$		
		$3\sqrt{5}$		$12\sqrt{3}$	$\div$	$3\sqrt{5}$	$=$	$\frac{4\sqrt{5}}{5}$
		$=$				$+$		
$4\sqrt{6}$	$\times$	$5\sqrt{5}$	$=$	$20\sqrt{30}$		$6\sqrt{5}$		

CHAPTER 11

Family Letter
Section B

What We Are Learning

Exponential Functions

VOCABULARY
These are the math words we are learning:

common ratio
the ratio by which successive terms of a geometric sequence differ

compound interest
interest earned or paid on *both* the principal and previously earned interest

exponential function
a function in which the independent variable appears as an exponent and is written in the form: $f(x) = ab^x$ where $a \neq 0$, $b \neq 1$, and $b > 0$

exponential decay
when a quantity decreases by the same rate, r, in each time period, t

exponential growth
when a quantity increases by the same rate, r, in the same time period, t

geometric sequence
the ratio of successive terms is the same number, r, called the common ratio

half-life
the time it takes for one-half of a substance to decay into another substance

Dear Family,

In this section, students investigate **sequences** and **exponential functions** using real-world data. Students will learn basic formulas to extend geometric sequences as well as finding rates of **exponential growth** and **exponential decay**.

Geometric sequences are similar to arithmetic sequences; however, geometric sequences are identified by a **common ratio**, r, rather than by constant first differences.

⇨ **Arithmetic Sequence: 3 5 7 9**

2 2 2

The constant first difference is 2.

⇨ **Geometric Sequence: 4 12 36 108**

÷3 ÷3 ÷3

The **common ratio** is 3.

Students can recognize geometric sequences if they find that the ratios of successive terms are constant.

Students will use the following formula to find the *n*th term in a geometric sequence.

$$a_n = a_1 r^{n-1}$$

a_1 = the first term
n = the nth term
r = the common ratio

Students will simply substitute known values and solve for a_n. It helps if students use a calculator to simplify the formula.

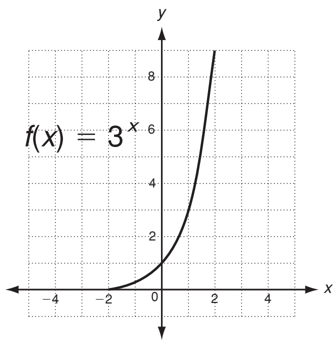
Find the 10th term in the geometric sequence 4, 12, 36, 108, …

$a_n = a_1 r^{n-1}$

$a_n = 4(3)^{10-1}$ Substitute 4 for a_1, 10 for n, and 3 for r.

$= 4(3)^9 = 78,732$

Students will also learn that exponential functions are functions that have a very specific form: $f(x) = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. The graph of an exponential function is a curve.



Notice the curve of the graph and how quickly it rises from left to the right.

To graph an exponential function, plot ordered pairs and connect them with a smooth curve.

Students will work with special exponential functions, such as those below, and use real-world data to calculate compound interest and investment growth. Remind students that these formulas are in the textbook as well as a number of examples, which they should review.

exponential growth	$y = a(1 + r)^t$	y = final amount a = original amount r = rate of increase t = time
compound interest	$A = P\left(1 + \frac{r}{n}\right)^{nt}$	A = balance after t years P = principal or original amount r = interest rate n = number of times a year interest is compounded t = time in years
exponential decay	$y = a(1 - r)^t$	y = final amount a = original amount r = rate of decrease t = time

After this section, students will now have a complete understanding of three types of functions: *linear*, *quadratic*, and *exponential*. Students will need to be able to distinguish between the three different functions and recognize the graphs and patterns that make these functions unique.

Sincerely,

Name ______________________ Date ____________ Class ____________

At-Home Practice

CHAPTER 11

Exponential Functions

Find the next three terms in each geometric sequence.

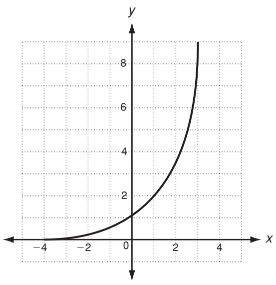
1. 4, 12, 36, 108, …

2. 1, −4, 16, −64, …

3. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

Match the graph to the correct exponential function.

A



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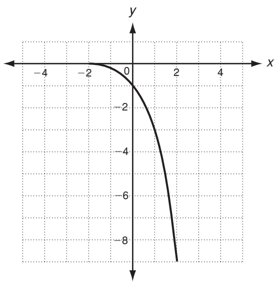

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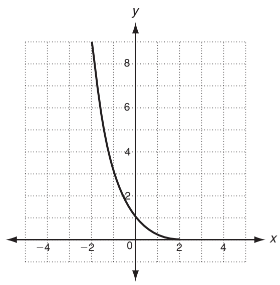
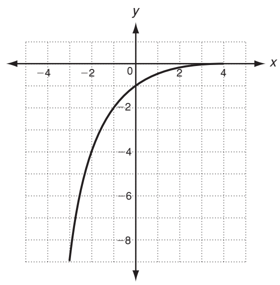


D



4. $y = -3^x$

5. $y = 2^x$

6. $y = -0.5^x$

7. $y = \left(\frac{1}{3}\right)^x$

Write a function to model each situation. Then find the value of the function after the given amount of time.

8. $3000 is invested at a rate of 6.25% compounded annually; 5 years

9. The population of a city was 32,325 in 2003 and is decreasing at a rate of about 1.5% per year; 4 years

Which kind of model best describes the data: linear, quadratic, exponential?

10. {(3, 4), (6, 8), (9, 12)}

11. {(1, 4), (2, 16), (2, 64)}

Answers: 1. 324, 972, 2916; **2.** 256, −1024, 4096; **3.** $\frac{1}{81}, \frac{1}{243}, \frac{1}{729}$; **4.** B; **5.** A; **6.** D; **7.** C; **8.** $y = 3000(1.0625)^t$; $4062.24; **9.** $y = 32{,}325(0.985)^t$; 30,429; **10.** linear; **11.** exponential

Name ______________________________ Date __________ Class __________

CHAPTER 11 Family Fun

Let's Make a Deal

Objective: To practice working with exponential growth formulas in everyday situations

Materials: number cubes
two calculators

Directions:
Your goal is to find the most profitable savings program.

Use the number cubes to help you to complete each column in the table below.

- To find the principal amount for each program, roll both number cubes. Find the product of the number cubes and multiply that product by 1000. All 5 savings programs will have this beginning principal amount.
- To determine how your money will be compounded, roll one number cube.

If you roll	**1**	**2**	**3**	**4, 5, or 6**
compound	daily	monthly	quarterly	yearly

- To find the investment time, roll both number cubes. Find the sum. This represents the time in years that your money is invested.

Complete the table.

Savings Program	**Principal Amount**	**Interest Rate**	**Compounded (daily, monthly, quarterly, or yearly)**	**Length of Time in Years**	**Ending Principal**
A		5.75%			
B		6.25%			
C		13.15%			
D		11.2%			
E		8.025%			

1. Which savings program yielded the most money? Why?

____

2. Which savings program yielded the least profit? Why?

____

3. If you could choose an account that compounded your money monthly or yearly, which would you choose? Explain your answer.

____

____

Possible answers: 1. the program with the highest interest rate. **2.** The program with the lowest interest rate. **3.** an account that compounds monthly; interest is earned on both the principal and previously-earned interest so the amount grows faster.