CHAPTER 10

Family Letter
Section A

What We Are Learning

Rational Functions and Expressions

VOCABULARY
These are the math words we are learning:

asymptote
a line that a graph gets closer to as the absolute value of the variable increases

discontinuous function
a rational function whose graph contains one or more holes, jumps, or breaks

excluded value
any x-value that makes the value of y undefined

inverse variation
a relationship in the form $y = \frac{k}{x}$, where k is a nonzero constant and $x \neq 0$.

rational expression
an algebraic expression whose numerator and denominator are polynomials

rational function
a function whose rule is a quotient of polynomials

Dear Family,

In this section, students will combine relationship concepts from previous sections and polynomial factoring skills to discover a new type of functional relationship, called a **rational function**.

Initially, students will learn to identify relationships that can be defined as **inverse variations**. In inverse variation, as one quantity increases, another quantity decreases and the product is constant. For instance, the faster you bike to the store, the shorter the time you spend biking. Time *varies inversely* with speed for a given distance.

Discuss with the student other scenarios in daily life that illustrate inverse variation. (*Think:* As one quantity goes up, the other goes down.)

Students will represent inverse variation as:

$$y = \frac{k}{x}$$

k is a non-zero constant.

x cannot equal 0.

or as $xy = k$

Inverse variations are introduced to students first because they are a simple example of a **rational function**. A rational function is a function whose rule is a quotient of polynomials, and a **rational expression** is simply a ratio of two polynomials.

Examples of Rational Expressions: $\frac{3x}{4 + x}$; $\frac{x^2 + 2x}{5x^2}$

Examples of Rational Functions: $y = \frac{3x}{4 + x}$; $y = \frac{x^2 + 2x}{5x^2}$

Note: The polynomial in the denominator must contain a variable (i.e. the degree of the polynomial must be at least 1). The expression $\frac{x}{5}$ is NOT a rational expression.

Make sure the student understands that division by 0 is undefined; you *cannot* divide a number by 0. This concept is fundamental in understanding rational expressions and functions.

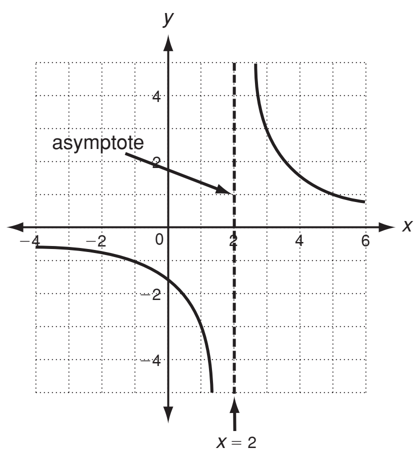
Values that cause the denominator to equal 0 are called **excluded values** and make the function undefined for that value.

For instance, when $x = -4$ the rational function $y = \frac{3x}{4 + x}$ is undefined because $y = \frac{3(-4)}{4 + (-4)} = \frac{-12}{0}$ and you cannot divide -12 by 0.

As students graph rational functions, they will discover that an *asymptote* occurs at the point of an excluded value(s). An **asymptote** is a line that the function approaches but never touches or crosses.

The student will learn to find excluded values by looking at the denominator of the function and determining the value(s) that make the denominator equal to 0. Once they determine these values, they can then easily draw the asymptote(s) on the graph.

Graph $\frac{3}{x - 2}$.



The **excluded value** for $\frac{3}{x - 2}$ is 2, so the function approaches but never touches the asymptote (the line $x = 2$).

Also in this section, students will learn to simplify rational expressions. This may seem familiar to students because it is similar to reducing fractions to lowest terms. Students will use previously learned skills, like factoring and exponent rules, to simplify rational expressions. To avoid making simple mistakes, encourage students to revisit those lessons in the textbook.

To see if the student has a good grasp on how to simplify rational expressions, ask the student to name for you the skill or the law he or she is using in each of the simplification steps.

Sincerely,

Name ______________________ Date ____________ Class ____________

CHAPTER 10

At-Home Practice

Rational Functions and Expressions

Tell whether each relationship represents an inverse variation.

1. $\frac{x}{y} = \frac{3}{4}$ ____________

2. $y = \frac{5}{x}$ ____________

3. $y = \frac{x}{9}$ ____________

4. $3x + y = 6$ ____________

5. $x = 8y$ ____________

6. $xy = -12$ ____________

Identify the excluded value and the vertical and horizontal asymptotes for each rational function.

7. $y = \frac{4}{x} - 6$ ____________

8. $y = \frac{5}{x} + 9$ ____________

9. $y = \frac{8}{x + 4}$ ____________

Find any excluded values of each rational expression.

10. $\frac{4x + 5}{x}$ ____________

11. $\frac{6}{x + 3}$ ____________

12. $\frac{2x - 4}{4x^2 - 2x}$ ____________

13. $\frac{-3}{x - 6}$ ____________

14. $\frac{x + 4}{x^2 - 4}$ ____________

15. $\frac{3x^2}{x^2 - x - 2}$ ____________

Simplify each rational expression, if possible. Identify any excluded values.

16. $\frac{5x^2}{20x^3}$ ____________

17. $\frac{6x}{4x^2 - x}$ ____________

18. $\frac{9 - 3x}{x^2 - 9}$ ____________

Answers: 1. no; **2.** yes; **3.** no; **4.** no; **5.** no; **6.** yes; **7.** excluded value: 0; $x = 0$; $y = -6$; **8.** excluded value: 0; $x = 0$; $y = 9$; **9.** excluded value: -4; $x = -4$; $y = 0$; **10.** 0 **11.** -3; **12.** 0 or $\frac{1}{2}$; **13.** 6; **14.** 2 or -2; **15.** -1 or 2; **16.** $\frac{1}{4x}$, 0; **17.** $\frac{6}{4x - 1}$, $\frac{1}{4}$ or 0; **18.** $\frac{-3}{x + 3}$; 3 or -3

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CHAPTER 10

Family Fun

Simply Excluded!

Objective: To simplify rational expressions and find the excluded values.

Materials: colored pencils

Directions:

- Simplify each rational expression.
- Find the excluded values for each expression
- Color the boxes in the table that contain those excluded values.
- Create your own rational expression in which the value(s) leftover in the table represent the expression's excluded value(s).

−2	4	8	1
−5	5	2	−2
−7	6	−3	0

1. $\frac{x + 5}{x^2 + 8x + 15}$ ____________

2. $\frac{2x - 3}{x^2 - 6x - 16}$ ____________

3. $\frac{(x + 2)}{(x^2 - 4)}$ ____________

4. $\frac{x^2 + x - 20}{x^2 - 9n + 20}$ ____________

5. $\frac{6x^2 + 15x}{3x^2 - 18x}$ ____________

6. $\frac{x + 7}{x^2 + 14x + 49}$ ____________

Leftover values: ____________ **New rational expression:** ____________

Answers: 1. $\frac{1}{x + 3}$; −3 or 5; **2.** $\frac{2x - 3}{(x + 2)(x - 8)}$; −2 or 8; **3.** $\frac{1}{x - 2}$; 2 or −2; **4.** $\frac{x + 5}{x - 5}$; 4 or 5; **5.** $\frac{2x + 5}{x - 6}$; 6 or 0; **6.** $\frac{1}{(x + 7)}$; −7; Leftover value: 1; Sample expression: $\frac{1}{x - 1}$

CHAPTER 10

Family Letter
Section B

What We Are Learning

Operations with Rational Expressions

VOCABULARY
These are the math words we are learning:

extraneous solution
a solution to a resulting equation that is not a solution to the original equation

rational equation
an equation that contains one or more rational expressions

Dear Family,

In this section, students will use previously learned skills in order to accurately add, subtract, multiply, and divide rational expressions.

At the onset of this section, the student will notice the similarities between performing mathematical operations on rational expressions and performing mathematical operations on fractions. For example, when students add or subtract rational expressions, they will find common denominators, just as they would if they were adding or subtracting fractions. Familiar steps also occur when multiplying or dividing rational expressions.

When students multiply or divide rational expressions, they may find it helpful to follow these simple steps:

STEPS FOR MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

1. Factor the numerators and the denominators.
2. Divide the numerators and denominators by the *greatest common factor* or *GCF*.
3. To multiply, multiply the numerators and then the denominators.
4. To divide, multiply by the reciprocal.
5. Simplify the numerators and the denominators by making sure the expression is in lowest terms.

The student may be surprised that the answer may not always be a whole number or integer, but another rational expression.

Be sure to have the student work through the examples in the textbook and refer back to the them when working the homework problems. There are numerous examples in the textbook that show the steps for each type of operation using various rational expressions. These example problems will help the students to realize the many different steps it may take to add, subtract, multiply, and/or divide rational expressions.

Encourage the student to revisit sections in the textbook that explain rational numbers or fractions if they have difficulty with the material in this section.

Once students learn to accurately perform mathematical operations on rational expressions, they will then be introduced to solving rational equations. A **rational equation** is an equation that contains one or more rational expressions.

Examples of Rational Equations:

$\frac{1}{x} = \frac{5}{x-2}$ $10 = \frac{1}{x+5}$ $\frac{x}{(x+2)(x-2)} = 7$

There are a few different ways to solve rational equations, but all methods lead to the same solution.

If the rational equation is a proportion, students may find it helpful to use the **Cross Product Property** to clear the fractions and then solve for the unknown variable. For example:

$\frac{1}{x} = \frac{5}{x-2}$

$1(x - 2) = x(5)$ Multiply diagonally across the = sign

$x - 2 = 5x$ Simplify

The student now has an easily-solvable equation.

Students may also solve a rational equation by using the *least common denominator*, or *LCD*, in order to clear the fractions. Students must remember to multiply **BOTH** sides of the equation by the LCD or they will not get the correct solution.

When students use the LCD method, they need to be careful of introducing an **extraneous solution**, a solution of the resulting equation that is not a solution of the original equation. It is imperative that students check their answers by substituting in the original equation. Students can then quickly determine if a solution is extraneous.

Students will need to work slowly and carefully to avoid simple math errors when operating on rational expressions. Encourage students to work a problem down the page to help keep the work organized and make it easier to find where a math error may have been made.

Sincerely,

Name ______________________ Date ____________ Class ____________

CHAPTER 10

At-Home Practice

Operations with Rational Expressions

Multiply. Simplify your answer.

1. $\frac{x+5}{x-3} \cdot (3x^2 - 9x)$

2. $\frac{16x^3y^4}{5xy} \cdot \frac{10x^4y^2}{24xy^2}$

3. $\frac{x^2 - 4x + 3}{x^2 - 5x + 4} \cdot \frac{x^2 - x - 12}{x^2 - 9}$

Divide. Simplify your answer.

4. $\frac{3xy^2}{4x^2y} \div \frac{9x^2y}{8xy}$

5. $\frac{2}{x^4} \div \frac{x+4}{x^6}$

6. $\frac{4x^2 + 16x + 12}{4x^2 + 8x + 4} \cdot \frac{6x + 3}{3x + 3}$

Add or subtract. Simplify your answer.

7. $\frac{8x+2}{3x} - \frac{(x-5)}{3x}$

8. $\frac{2x}{5x^2} + \frac{4}{x}$

9. $\frac{4x}{x-5} + \frac{3x+4}{x-5}$

10. $\frac{x^2 + 7x + 6}{x^2 - 36} - \frac{6}{x-6}$

Divide. Check your answer.

11. $(4x^2 + 18x) \div 2x$

12. $(35x^4 - 14x^3 - 56x) \div (-7x)$

13. $(6x^2 + 13x + 5) \div (2x + 1)$

Solve. Check your answer.

14. $\frac{2}{x+1} = \frac{3}{x+2}$

15. $\frac{4x}{6} = \frac{2x+6}{12} + 4$

16. $\frac{x-5}{x^2-1} + \frac{2x}{x-1} = 1$

Answers: 1. $3x^2 + 15x$; **2.** $\frac{4x^5y^3}{3}$; **3.** 1; **4.** $\frac{2y}{3x^2}$; **5.** $\frac{2x^2}{x+4}$; **6.** $\frac{x+3}{2x+1}$; **7.** $\frac{7x+7}{3x}$; **8.** $\frac{22}{5x}$; **9.** $\frac{7x+4}{x-5}$; **10.** $\frac{x-5}{x-6}$; **11.** $2x + 9$; **12.** $-5x^3 + 2x^2 + 8$; **13.** $3x + 5$; **14.** $x = 1$; **15.** $x = 9$; **16.** $x = -4$

Name ______________________________ Date __________ Class __________

CHAPTER 10

Family Fun

Missing Values

Objective: To practice simplifying rational expressions

Directions:

Find the missing values in each rational expression using the numbers in the box. The numbers may be used more than once.

2	3	4	5	6	7	8	10	13	16	20	32

Add.

$\frac{x}{5x^2} + \frac{2}{3x}$

1. $\frac{x}{5x^2}\left(\frac{\square}{\square}\right) + \frac{2}{3x}\left(\frac{\square x}{\square x}\right)$
2. $\frac{\square x}{15x^2} + \frac{\square x}{15x^2}$
3. $\frac{\square x}{15x^2}$
4. $\frac{\square}{15x}$

Subtract.

$\frac{x^2 + 10x}{x^2 + 2x - 8} - \frac{4}{x - 2}$

5. $\frac{x^2 + 10x}{x^2 + 2x - 8} - \frac{4}{x - 2}\left(\frac{x + \square}{x + \square}\right)$
6. $\frac{x^2 + 10x}{x^2 + 2x - 8} - \frac{(4x + \square)}{x^2 + 2x - 8}$
7. $\frac{x^2 + 10x - 4x - \square}{x^2 + 2x - 8}$
8. $\frac{x^2 + \square x - \square}{x^2 + 2x - 8}$
9. $\frac{(x + \square)(x - 2)}{(x + \square)(x - 2)}$
10. $\frac{x + \square}{x + \square}$

Multiply.

$\frac{6x^3y^2}{3y} \cdot \frac{12xy}{9x^2}$

11. $\square x^3y \cdot \frac{\square y}{\square x}$
12. $\frac{\square x^{\square}y^{\square}}{3}$

Divide.

$\frac{24x^3y^2z^4}{5y^4z^2} \div \frac{3x^2z^4}{20x^2y^2x^3}$

13. $\frac{24x^3y^2z^4}{5y^4z^2} \cdot \frac{\square x^2y^2x^3}{3x^2z^4}$
14. $\frac{\square x^{\square}y^{\square}z^{\square}}{x^2y^4z^6}$
15. $\square x^{\square}z$

Answers: 1. $\frac{3}{3}$; $\frac{5}{5}$ **2.** 3; 10 **3.** 13 **4.** 13 **5.** $\frac{4}{4}$ **6.** 16 **7.** 16 **8.** 6; 16 **9.** 8; 4 **10.** 8; 4 **11.** 2; $\frac{4}{3}$ **12.** 8; 2; 2
13. 20 **14.** 32; 5; 4; 7 **15.** 32; 3