

Chapter 14 Parent Guide Further Topics in Trigonometry

Chapter 14 develops the use of the trigonometric functions to solve more complicated problems. Students will use formulas, such as the law of sines, the law of cosines, and trigonometric identities to solve problems.

Students who intend to study physics and calculus should become competent in manipulating trigonometric identities and using trigonometric formulas.

Chapter 14 begins with a study of the law of sines and the law of cosines. Then it proves fundamental trigonometric identities and uses them to rewrite expressions. Students will study identities that involve adding and subtracting angles as well as doubling and halving angles. The chapter ends with solving trigonometric equations, which involves using identities and factoring to rewrite the equations.

Lesson 14.1 uses the law of sines to solve problems. Lesson 14.2 uses the law of cosines to solve problems. Lesson 14.3 lists and uses the six fundamental trigonometric identities to rewrite expressions. Lesson 14.4 evaluates expressions by using the sum and difference identities. Lesson 14.5 evaluates and simplifies expressions by using the double-angle and half-angle identities. Students solve trigonometric equations to solve real-world problems in Lesson 14.6.

You can do the following activity with your child to show the importance of trigonometry when solving real-world problems involving angles and one side of a triangle. You will need a scientific or graphics calculator for this activity.

PROBLEM FOR DISCUSSION (See textbook page 886)

The triangular piece of land represented in the diagram on page 886 will be used for a new park. What is the approximate area of the land?

1. Discuss what information is given and what information is still needed to find the area of the triangle. Once you get the missing piece of information, what formula will you use to solve the problem?

To find the area of a triangle, you need the length of the base and the altitude. For this triangle, you do not have the altitude.

Once you determine the altitude, use the formula for the area of a triangle, $A = \frac{1}{2}bh$, where b is the base and h is the altitude.

2. Discuss how to use the definition of sine to determine BD , the height of $\triangle ABC$. Determine which two pieces of information can be used to find BD . Explain why $\triangle ABC$ does not need to be a right triangle to determine its area.

The sine of an angle is equal to the ratio of the side opposite the angle to the length of the hypotenuse.

The two pieces of information needed are $m\angle C = 32^\circ$ and $BC = 2$ miles.

The triangle does not have to be a right triangle. You just need to be able to find the altitude.

3. Discuss what two pieces of information will be used to determine the area. What is the area of the triangle?

Side BD and side AC will be used to find the area of triangle ABC .

$$A = \frac{1}{2} bh$$

$$A = \frac{1}{2} (AC)(BD) \quad AC = 2.5 \quad \sin 32 = \frac{BD}{BC}$$
$$BC \sin 32 = BD$$

$$A = \frac{1}{2} (2.5)(BD) \quad 2 \sin 32 = BD$$

$$A = \frac{1}{2} (2.5)(2 \sin 32)$$

$$A = 1.3 \text{ square miles}$$

3. To use the Pythagorean theorem to solve this problem, what information is needed? Why is that piece of information impossible to get?

To use the Pythagorean theorem, the lengths of BC , and DC are needed. You have no way of knowing what DC is because that information was not given.

4. What have you learned about the sine function from this activity?

Area formulas can be used to derive the law of sines. The law of sines can be used to find the measures of a triangle.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 14.1

$$13. K = \frac{1}{2}bc \sin A$$

$$= \frac{1}{2}(7)(10) \sin 45^\circ$$

$$\approx 24.7 \text{ cm}^2$$

$$25. \frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 25^\circ}{c} = \frac{\sin 50^\circ}{15}$$

$$c = \frac{15 \sin 25^\circ}{\sin 50^\circ}$$

$$c \approx 8.3$$

52. a. Let x = distance from A to the fire
and F = angle at fire.

$$F = 180^\circ - 65.23^\circ - 56.47^\circ = 58.3^\circ$$

$$\frac{\sin 58.3^\circ}{10} = \frac{\sin 56.47^\circ}{x}$$

$$x \approx 9.8$$

The distance from A to the fire is 9.8 kilometers.

- b. Let y = distance from B to the fire.

$$\frac{\sin 65.23^\circ}{y} = \frac{\sin 58.3^\circ}{10}$$

$$y \approx 10.7$$

The distance from B to the fire is 10.7 kilometers.

Lesson 14.2

10. SAS information is given.

$$b^2 = a^2 + c^2 - 2bc \cos A$$

$$b = \sqrt{5^2 + 13^2 - 2(5)(13) \cos 95^\circ}$$

$$b \approx 14.3$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{5} = \frac{\sin 95^\circ}{14.3}$$

$$\sin A = \frac{5 \sin 95^\circ}{14.3}$$

$$\sin A \approx 0.3483$$

$$A \approx 20.4$$

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19. Solve for the largest angle first, in case it is obtuse.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos B = \frac{49^2 - 35^2 - 45^2}{-2(35)(45)}$$

$$\cos B \approx 0.2695$$

$$B \approx 74.4^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{a \sin B}{b}$$

$$\sin A = \frac{35 \sin 74.4^\circ}{49}$$

$$\sin A \approx 0.6880$$

$$A \approx 43.5^\circ$$

$$C \approx 180^\circ - A - B$$

$$C \approx 180^\circ - 43.5^\circ - 74.4^\circ$$

$$C \approx 62.1^\circ$$

Thus, $A \approx 43.5^\circ$, $B \approx 74.4^\circ$, and $C \approx 62.1^\circ$.

28. SAS information is given.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = \sqrt{123^2 + 63.2^2 - 2(123)(63.2) \cos 114^\circ}$$

$$a \approx 159.5$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{123} = \frac{\sin 114^\circ}{159.5}$$

$$\sin B \approx 0.7045$$

$$B \approx 44.8^\circ$$

$$C \approx 180^\circ - A - B$$

$$C \approx 180^\circ - 114^\circ - 44.8^\circ$$

$$C \approx 21.2^\circ$$

Thus, $a \approx 159.5$, $B \approx 44.8^\circ$, and $C \approx 21.2^\circ$.

37. SSA information is given.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{8} = \frac{\sin 58^\circ}{10}$$

$$\sin B \approx 0.6784$$

$$B \approx 42.7^\circ \text{ or } B \approx 180^\circ - 42.7^\circ = 137.3^\circ \text{ (not possible, since } A = 58^\circ)$$

$$C = 180^\circ - A - B$$

$$C \approx 180^\circ - 58^\circ - 42.7^\circ$$

$$C \approx 79.3^\circ$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$c = \frac{b \sin C}{\sin B}$$

$$c \approx \frac{8 \sin 79.3^\circ}{\sin 42.7^\circ}$$

$$c \approx 11.6$$

Thus, $B \approx 42.7^\circ$, $C \approx 79.3^\circ$, and $c \approx 11.6$.

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Lesson 14.3

$$\begin{aligned} 10. \csc \theta &= \frac{r}{y} = \frac{1}{\frac{y}{r}} \\ &= \frac{1}{\sin \theta}, \sin \theta \neq 0 \end{aligned}$$

$$18. \sec \theta \cos^2 \theta = \frac{1}{\cos \theta} \cdot \cos^2 \theta = \cos \theta$$

$$\begin{aligned} 28. (1 - \sin^2 \theta)(1 + \sec^2 \theta) &= \cos^2 \theta \left(1 + \frac{1}{\cos^2 \theta}\right) \\ &= \cos^2 \theta + \frac{\cos^2 \theta}{\cos^2 \theta} \\ &= \cos^2 \theta + 1 \end{aligned}$$

$$42. \frac{\csc \theta}{\sin \theta} = \frac{\frac{1}{\sin \theta}}{\sin \theta} = \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

Lesson 14.4

$$\begin{aligned} 15. \sin(135^\circ + 180^\circ) &= \sin 135^\circ \cos 180^\circ + \cos 135^\circ \sin 180^\circ \\ &= \frac{\sqrt{2}}{2}(-1) + \left(-\frac{\sqrt{2}}{2}\right)(0) = -\frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} 25. \cos(90^\circ + \theta) &= \cos 90^\circ \cos \theta - \sin 90^\circ \sin \theta \\ &= 0 \cdot \cos \theta - 1 \cdot \sin \theta \\ &= -\sin \theta \end{aligned}$$

30. Answers may vary. Sample answer:

Let $A = 30^\circ$ and $B = 60^\circ$.

$$\cos(A + B) = \cos(30^\circ + 60^\circ)$$

$$= \cos 90^\circ = 0$$

$$\cos A + \cos B = \cos 30^\circ + \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}$$

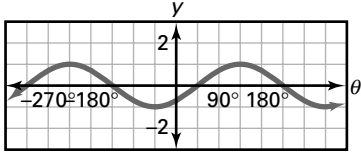
$$= \frac{\sqrt{3} + 1}{2}$$

So, $\cos(A + B) \neq \cos A + \cos B$

$$\begin{aligned} 37. \sin 15^\circ &= \sin(60^\circ - 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

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54. $y = \sin(\theta - 45^\circ)$



$$\begin{aligned}
 61. & \begin{bmatrix} \cos 135^\circ & -\sin 135^\circ \\ \sin 135^\circ & \cos 135^\circ \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} \frac{3\sqrt{2}}{2} - \frac{2\sqrt{2}}{2} \\ -\frac{3\sqrt{2}}{2} - \frac{2\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{5\sqrt{2}}{2} \end{bmatrix} \\
 &P\left(\frac{\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)
 \end{aligned}$$

Lesson 14.5

10. $\cos 2\theta + 2 \sin^2 \theta = \cos^2 \theta - \sin^2 \theta + 2 \sin^2 \theta$
 $= \cos^2 \theta + \sin^2 \theta = 1$

15. $\sin 4\theta = \sin(2\theta + 2\theta)$
 $= \sin 2\theta \cos 2\theta + \cos 2\theta \sin 2\theta$
 $= 2 \sin 2\theta \cos 2\theta$
 $= 2(2 \sin \theta \cos \theta)(\cos^2 \theta - \sin^2 \theta)$
 $= 4 \sin \theta \cos \theta (\cos^2 \theta - \sin^2 \theta)$
 $= 4 \sin \theta \cos^3 \theta - 4 \sin^3 \theta \cos \theta$

26. $x = 2, y = -\sqrt{21}, r = 5; \sin \theta = -\frac{\sqrt{21}}{5}, \cos \theta = \frac{2}{5}$

$$\begin{aligned}
 \sin 2\theta &= 2 \sin \theta \cos \theta & \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\
 &= 2\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right) & &= \left(\frac{2}{5}\right)^2 - \left(\frac{\sqrt{21}}{5}\right)^2 \\
 &= -\frac{4\sqrt{21}}{25} & &= \frac{4}{25} - \frac{21}{25} = -\frac{17}{25}
 \end{aligned}$$

35. $x = -\sqrt{11}, y = 5, r = 6; \sin \theta = \frac{5}{6}, \cos \theta = -\frac{\sqrt{11}}{6}$

$$\begin{aligned}
 \sin \frac{\theta}{2} &= \sqrt{\frac{1 - \cos \theta}{2}} & \cos \frac{\theta}{2} &= \sqrt{\frac{1 + \cos \theta}{2}} \\
 &= \sqrt{\frac{1 - \left(-\frac{\sqrt{11}}{6}\right)}{2}} & &= \sqrt{\frac{1 + \left(-\frac{\sqrt{11}}{6}\right)}{2}} \\
 &= \sqrt{\frac{1}{2}\left(\frac{6 + \sqrt{11}}{6}\right)} & &= \sqrt{\frac{1}{2}\left(\frac{6 - \sqrt{11}}{6}\right)} \\
 &= \sqrt{\frac{6 + \sqrt{11}}{12}} = \sqrt{\frac{1}{2} + \frac{\sqrt{11}}{12}} & &= \sqrt{\frac{6 - \sqrt{11}}{12}} = \sqrt{\frac{1}{2} - \frac{\sqrt{11}}{12}}
 \end{aligned}$$

Lesson 14.6

11. $4 \cos \theta - 2 = 0$

$4 \cos \theta = 2$

$\cos \theta = \frac{1}{2}$

$\theta = 60^\circ, 300^\circ$

Thus, $\theta = 60^\circ + n360^\circ$ or

$\theta = 300^\circ + n360^\circ$.

16. $\tan \theta - \sqrt{3} = 0$

$\tan \theta = \sqrt{3}$

$\theta = 60^\circ, 240^\circ$

Thus, $\theta = 60^\circ + n360^\circ$ or $\theta = 240^\circ + n360^\circ$.

21. $2 \sin^2 \theta = 1 - \sin \theta$

$2 \sin^2 \theta + \sin \theta - 1 = 0$

Let $u = \sin \theta$.

$2u^2 + u - 1 = 0$

$(2u - 1)(u + 1) = 0$

$2u - 1 = 0$ or $u + 1 = 0$

$u = \frac{1}{2}$ or $u = -1$

$\sin \theta = \frac{1}{2}$ or $\sin \theta = -1$

$\theta = 30^\circ, 150^\circ$ or $\theta = 270^\circ$

Thus, $\theta = 30^\circ, 150^\circ, \text{ or } 270^\circ$.

27. $\cos^2 \theta + \cos \theta = 0$

$\cos \theta(\cos \theta + 1) = 0$

$\cos \theta = 0$ or $\cos \theta + 1 = 0$

$\theta = 90^\circ, 270^\circ$ or $\cos \theta = -1$

$\theta = 180^\circ$

Thus, $\theta = 90^\circ, 180^\circ \text{ or } 270^\circ$.

52. $y = 5 \cos(\pi t)$

$5 = 5 \cos(\pi t)$

$1 = \cos(\pi t)$

$\pi t = 0$ or $\pi t = 2\pi$

$t = 0$ or $t = 2$

The weight is 5 centimeters above the rest position at $0 + 2n$ seconds.