

Chapter 13 Parent Guide Trigonometric Functions

Trigonometry is a combination of algebra and geometry. It is used to determine distances that are difficult to measure physically. Such distances include the width of a river, the distance between two mountain tops, the height of a waterfall, and the height of a tree.

Someone wanting to cut down a tree, for example, might want to know its height to determine how close it will come to a building structure when it falls.

Trigonometry is also used in professions involving housing construction, aviation, road and bridge construction, electronics, and plumbing, to name a few.

Both Chapters 13 and 14 have to do with trigonometry. Chapter 13 involves definitions and graphs of trigonometric functions. It starts out with a review of basic trigonometric identities and angle

rotation. From there, students will evaluate trigonometric functions for certain angle values, use radian measure to determine arc length, graph trigonometric functions, and use inverses of trigonometric functions.

Lesson 13.1 evaluates trigonometric functions of acute angles. Lesson 13.2 discusses angles rotated clockwise and counterclockwise about the origin. Lesson 13.3 finds exact and approximate values for trigonometric functions of any angle. Lesson 13.4 shows how to convert from degree measure to radian measure and vice versa. Lesson 13.5 relates graphs of sine, cosine, and tangent functions. Lesson 13.6 evaluates expressions involving the inverses of the sine, cosine, and tangent functions.

You can help your child develop basic concepts for this chapter by doing the following activity together.

PROBLEM FOR DISCUSSION (See textbook page 836)

You can use angles of rotation to describe the rate at which an airplane propeller rotates.

In geometry, an angle is defined by two rays that have a common endpoint. How is the trigonometric definition of an angle different from the geometric definition?

1. Name the types of angles used in geometry. What is the maximum measure that an angle can be? Can the measure of a geometric angle be less than zero degrees?

Some of the angles used in geometry are obtuse, acute, right, and straight.

The maximum measure that an angle can be in geometry is 180° .

No, the measure of a geometric angle can not be less than zero degrees because in geometry concrete objects have degree measures. For instance, triangles and polygons have degree measures. It doesn't make any sense for a triangle to have negative angle measures.

2. Discuss the angles in trigonometry. Looking at the angle diagrams on page 837, do trigonometric angles have a maximum or minimum? Justify your answer.

Angles in trigonometry are based on the angles of the unit circle. Trigonometric angles do not have a maximum or minimum because the measure can keep going around in a circle. Trigonometric angles, however, do repeat. For instance, an angle measure of 360° is the same as a degree measure of 720° .

3. Discuss how trigonometric angles differ from geometric angles.

Trigonometric angles can be negative, and they do not have a maximum or a minimum. Geometric angles cannot be negative and they have a maximum of 180° and a minimum of 0° .

4. Discuss why 150° is the same as -210° in trigonometry.

The two measures are the same because if you start at 0° and go counterclockwise 150° , you end up in the same place as if you had started at 0° and gone clockwise 210° .

5. What have you learned about trigonometric angles from this activity?

This activity demonstrated the difference between geometric angles and trigonometric angles and discussed the characteristics of trigonometric angles.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 13.1

10. $\cos J = \frac{8}{17} \approx 0.4706$

24. $\tan G = \frac{3}{2} = 1.5$

32. $\tan A = \frac{5}{7}$

$m\angle A = \tan^{-1} \frac{5}{7} \approx 35.5^\circ$

39. $\cos R = \frac{3.3}{3.9}$

$m\angle R = \cos^{-1} \frac{3.3}{3.9} \approx 32^\circ$

$m\angle S = 90^\circ - m\angle R \approx 90^\circ - 32^\circ \approx 58^\circ$

$3.9^2 = ST^2 + 3.3^2$

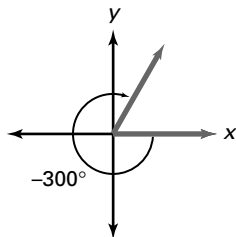
$ST = \sqrt{3.9^2 - 3.3^2} \approx 2.1$

47. $\tan A = \frac{6.5}{186}$

$m\angle A = \tan^{-1} \frac{6.5}{186} \approx 2^\circ$

Lesson 13.2

11.



29. $410^\circ - 360^\circ = 50^\circ$

$410^\circ - 720^\circ = -310^\circ$

$\theta_{\text{ref}} = 50^\circ$

38. $x = -3; y = -2$

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-2)^2} = \sqrt{13}$

$\sin \theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$

$\csc \theta = \frac{r}{y} = -\frac{\sqrt{13}}{2}$

$\cos \theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$

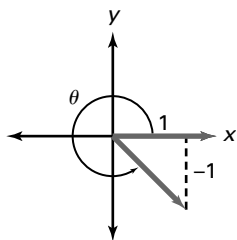
$\sec \theta = \frac{r}{x} = -\frac{\sqrt{13}}{3}$

$\tan \theta = \frac{y}{x} = \frac{2}{3}$

$\cot \theta = \frac{x}{y} = \frac{3}{2}$

Chapter 13

50. In Quadrant IV, $\tan \theta = -1$.



$$x = 1; y = -1$$

$$r^2 = x^2 + y^2 \\ = 1^2 + (-1)^2 = 2$$

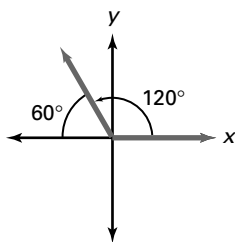
$$r = \sqrt{2}$$

$$\csc \theta = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

61. $\frac{720^\circ}{360^\circ} = 2$ rotations counterclockwise

Lesson 13.3

11. Draw a diagram and find the reference angle.



$$\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$$

$$\tan 120^\circ = -\tan 60^\circ = -\sqrt{3}$$

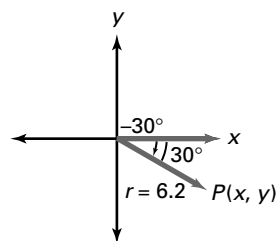
26. Draw a diagram and find the reference angle.

$$\theta = -30^\circ, r = 6.2$$

$$x = r \cos \theta = 6.2 \cos(-30^\circ) = 6.2 \cos 30^\circ = 6.2 \left(\frac{\sqrt{3}}{2}\right) = 3.1\sqrt{3}$$

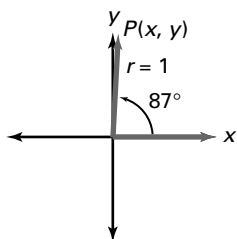
$$y = r \sin \theta = 6.2 \sin(-30^\circ) = -6.2 \sin 30^\circ = -6.2 \left(\frac{1}{2}\right) = -3.1$$

$$P(3.1\sqrt{3}, -3.1)$$



Chapter 13

34. Draw a diagram and find the reference angle.



$$\begin{aligned}\theta &= 87^\circ, r = 1 \\ x &= r \cos \theta = 1 \cos 87^\circ \approx 0.05 \\ y &= r \sin \theta = 1 \sin 87^\circ \approx 1.00 \\ P &(0.05, 1.00)\end{aligned}$$

45. $\theta = -495^\circ$

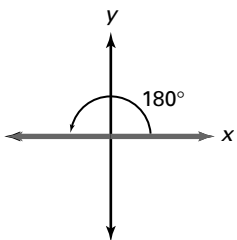
$$-495^\circ + 2(360^\circ) = 225^\circ$$

$$\sin 225^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\cos 225^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\tan 225^\circ = \tan 45^\circ = 1$$

60. Draw a diagram and find the reference angle.



$$\tan 180^\circ = \tan 0^\circ = 0$$

95. $\cos 60^\circ = \frac{JL}{30}$

$$\frac{1}{2} = \frac{JL}{30}$$

$$2JL = 30$$

$$JL = 15 \text{ centimeters}$$

$$\sin 60^\circ = \frac{KL}{30}$$

$$\frac{\sqrt{3}}{2} = \frac{KL}{30}$$

$$2KL = 30\sqrt{3}$$

$$KL = 15\sqrt{3} \text{ centimeters}$$

$$m\angle K = 90^\circ - 60^\circ = 30^\circ$$

Chapter 13

Lesson 13.4

15. $270^\circ \cdot \frac{\pi}{180^\circ} = \frac{3\pi}{2}$ radians

31. $\frac{-\pi}{4} \cdot \frac{180^\circ}{\pi} = -45^\circ$

40. $\frac{-\pi}{6} \cdot \frac{180^\circ}{\pi} = -30^\circ$
 $\sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

56. $s = r\theta$
 $= (5) 4.28$
 $= 21.4$ meters

65. Area of the circle $= \pi r^2 = \pi(12)^2 = 144\pi$ in²
Let x be the central angle for the sector.

$$\frac{x}{2\pi} = \frac{55.5 \text{ in}^2}{144\pi \text{ in}^2}$$
$$x = \frac{55.5(2\pi)}{144\pi}$$
$$= \frac{55.5}{72} = \frac{37}{48}$$

The central angle is $\frac{37}{48}$ radian.

68. $C = 2\pi r$
 $= 2\pi \cdot 3$
 $= 6\pi$ centimeters
distance per minute $= 200(6\pi)$
 $= 1200\pi$
 ≈ 3770 centimeters

The speed at a point on the outer edge is about 3770 centimeters per minute.

Lesson 13.5

13. $y = 4.5 \tan 3\theta$
amplitude: none
period: $\frac{\pi}{3}$

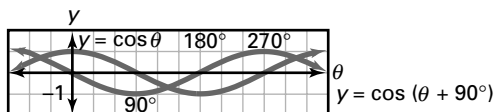
17. $y = 3 \cos(\theta + 90^\circ)$
amplitude: 3
period: 2π

23. $y = \sin(\theta + 60^\circ) + 1$
phase shift: 60° to the left
vertical translation: up 1

Chapter 13

33. $y = \cos(\theta + 90^\circ)$

The parent function is translated 90° to the left.



54. The air conditioner runs until it reaches the desired temperature. The air conditioner takes 12 minutes to reach the minimum temperature after being turned on.

Lesson 13.6

14. $\tan 150^\circ = -\frac{\sqrt{3}}{3}$ $\tan 330^\circ = -\frac{\sqrt{3}}{3}$

All possible values of $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ are

$150^\circ + n360^\circ$ and $330^\circ + n360^\circ$, where n is an integer.

22. $\sin 45^\circ = \frac{\sqrt{2}}{2}$

$$\sin^{-1} \frac{\sqrt{2}}{2} = 45^\circ$$

41. $\tan 225^\circ = 1$
 $\cos^{-1} 1 = 0^\circ$

49. $x = \frac{1}{4}y$

$$\cos \theta = \frac{x}{y} = \frac{\frac{1}{4}y}{y} = \frac{1}{4}$$

$$\theta = \cos^{-1} \frac{1}{4} \approx 75.5^\circ$$

The angle formed by the ladder and the ground is about 75.5° .