

Chapter 10 **Parent Guide**
Discrete Mathematics: Counting Principles and Probability

Probability is expressed as a ratio of the number of actual occurrences expected to the total number of possible occurrences. Counting sets and subsets is imperative for finding probability. In Chapter 10, students will study permutations and combinations, and find probabilities of dependent and independent events.

You may recall your child studying probability in other courses. This is because probability is an important field of study that requires logical thinking and problem-solving skills that differ from other areas of mathematics. Understanding probability will help your child to grow mathematically.

Probability is used in a wide variety of fields including game theory, naturalists studying the growth of a plant or animal population, insurance actuaries, and law enforcement.

The chapter first develops simple probability concepts and then moves on to permutations and combinations, two counting methods used in probability. The chapter then focuses on finding probabilities of independent and dependent events.

Lesson 10.1 finds the theoretical probability of a simple event and uses the Fundamental Counting Principle. Lesson 10.2 solves problems involving permutations. Lesson 10.3 solves problems involving combinations. Lesson 10.4 introduces finding two-event probabilities. Lesson 10.5 involves finding the probability of two or more independent events. Lesson 10.6 explores conditional probabilities. Lesson 10.7 concludes the chapter with experimental probability and simulation.

You may want to help your child develop the concept of permutations by doing the following activity together.

PROBLEM FOR DISCUSSION (See textbook page 636)

There are many situations that involve an ordered arrangement, or permutation, of objects. For example, 12-tone music, developed by Arnold Schoenberg, consists of permutations of all 12 tones in an octave. Each of the 12 notes must be used exactly once before any are repeated. How can you count the total number of possible arrangements?

1. Order the letters A and B in as many different ways as possible. Then order the letters A , B , and C in as many different ways as possible. How many ways can you order two letters? How many ways can you order three letters?

The letters A and B can be ordered as AB or as BA .

The letters A , B and C can be ordered as ABC , ACB , BAC , BCA , CBA , and CAB .

You can order two letters in two different ways.

You can order three letters in six different ways.

3. In ordering A , B , and C , how many choices do you have to select the first letter? After choosing it, how many are left to select the second letter? After the first two letters are selected, how many ways are left to select the last letter? How do these numbers relate to the number of ways to order three letters?

You have three choices to select the first letter. These choices are A , B , or C . For example, say you choose A .

After choosing it, you have two letters to choose from for the second letter. In the example, these two letters are B and C . Say you choose C .

After choosing the first two letters, you only have one left for the third letter. This would be B .

First, you know there are 3 choices for the first letter. For each of the 3 choices there are two choices for other 2 letters. This gives 6 different ways to order the 3 letters.

3. Discuss how to find the number of ways to order four letters without listing them. Then calculate the number. Does it match the answer calculated on page 636?

To order the four letters, you know that there are four options for the first letter.

For the second letter there are 3 different choices. Now, you can use the information learned in Exercises 1 and 2. You know that three letters can be ordered 6 different ways.

For each of the 4 choices for the first letter, there are 6 choices for the next 3 letters. This gives a total of 24 different ways to order the letters. This matches the textbook calculation.

The following are complete worked out solutions to selected exercises in the student textbook.

These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Chapter 10

$$27. {}_8P_4 = \frac{8!}{(8-4)!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 1680$$

40. The word *pencil* contains 6 letters, all different. So there are $6! = 720$ permutations.

$$51. \text{ a. } \begin{array}{cccccccc} & \text{1st} & & \text{2nd} & & \text{3rd} & & \text{4th} \\ & \text{digit} & & \text{digit} & & \text{digit} & & \text{digit} \\ & 10 & \times & 10 & \times & 10 & \times & 10 & = & 10,000 \end{array}$$

There are 10,000 possible PIN numbers.

$$\text{ b. } \begin{array}{cccccccc} & \text{1st} & & \text{2nd} & & \text{3rd} & & \text{4th} \\ & \text{digit} & & \text{digit} & & \text{digit} & & \text{digit} \\ & 10 & \times & 9 & \times & 8 & \times & 7 & = & 5040 \end{array}$$

There are 5040 possible PIN numbers.

Lesson 10.3

$$13. {}_{11}C_1 = \frac{11!}{1!(11-1)!} = \frac{11 \times 10!}{1(10!)} = 11$$

$$22. {}_{12}C_8 = \frac{12!}{8!(12-8)!} = \frac{12!}{8!4!} = \frac{12 \times 11 \times 10 \times 9 \times 8!}{8!(4 \times 3 \times 2 \times 1)} = \frac{990}{2} = 495 \text{ ways}$$

$$\begin{aligned} 27. {}_3C_1 \times {}_9C_4 &= \frac{3!}{1!2!} \times \frac{9!}{4!5!} \\ &= \frac{3 \times 2!}{2!} \times \frac{9 \times 8! \times 7 \times 6 \times 5!}{(4 \times 3 \times 2 \times 1)(5!)} \\ &= \frac{3}{1} \times \frac{378}{3} = 378 \text{ ways} \end{aligned}$$

36. Permutation, because the contestants receive different prizes.

Lesson 10.4

11. There are 3 favorable outcomes (1, 2, or 3), so probability = $\frac{3}{6} = \frac{1}{2}$.

22. The events are inclusive, because 5 and 6 are both greater than 4 and less than 7.

$$\begin{aligned} P(\text{greater than 4 or less than 7}) &= P(\text{greater than 4}) + P(\text{less than 7}) - P(\text{greater than 4 and less than 7}) \\ &= \frac{30}{36} + \frac{15}{36} - \frac{9}{36} \\ &= \frac{36}{36} = 1 \end{aligned}$$

$$32. P(E^c) = 1 - P(E) = 1 - 0.782 = 0.218$$

Chapter 10

42. Total area = $10 \times 10 = 100$; area of smaller square = $6 \times 6 = 36$;
area of circle = $\pi(3)^2 = 9\pi$; area of composite figure = $36 + \frac{1}{2}(9\pi) = 36 + \frac{9}{2}\pi$;
area of overlap = $\frac{1}{2}9\pi = \frac{9}{2}\pi$
- a. probability = $\frac{\text{area of circle}}{\text{total area}} = \frac{9\pi}{100} \approx 0.283 = 28.3\%$

Lesson 10.5

8. $P(A \text{ and } C) = P(A) \times P(C) = 0.5 \times 0.75 = 0.375$
15. $P(\text{less than } 5) = P(1, 2, 3 \text{ or } 4) = \frac{4}{6} = \frac{2}{3}$
 $P(6) = \frac{1}{6}$
 $P(\text{less than } 5 \text{ and } 6) = 0$, since 6 is not less than 5.
 $P(\text{less than } 5) \times P(6) = \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$
Since $0 \neq \frac{1}{9}$, the events are not independent;
they are dependent.
19. Let A = the number is a 5 or less than 7. Then A^c = the number is *not* a 5, nor less than 7.

$$P(A) = P(1, 2, 3, 4, 5, \text{ or } 6) = \frac{6}{8} = \frac{3}{4}$$

$$P(A^c) = 1 - \frac{3}{4} = \frac{1}{4}$$

Since each event is independent of the others,

$P(\text{exactly one number is a 5 or less than } 7)$

$$\begin{aligned} & \begin{array}{ccccccccc} \text{1st} & \text{2nd} & \text{3rd} & \text{1st} & \text{2nd} & \text{3rd} & \text{1st} & \text{2nd} & \text{3rd} \\ \text{spin} & \text{spin} & \text{spin} & \text{spin} & \text{spin} & \text{spin} & \text{spin} & \text{spin} & \text{spin} \end{array} \\ & = P(A) \times P(A^c) \times P(A^c) + P(A^c) \times P(A) \times P(A^c) + P(A^c) \times P(A^c) \times P(A) \\ & = \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \\ & = 3\left(\frac{3}{64}\right) = \frac{9}{64} \approx 0.141 = 14.1\% \end{aligned}$$

24. $P(\text{escape contacts and camera}) = P(\text{escape contacts}) \times P(\text{escape camera})$
 $= 0.4 \times 0.6 = 0.24 = 24\%$

Lesson 10.6

12. $P(\text{blue first}) = \frac{5}{22}$
 $P(\text{red second}) = \frac{8}{21}$
 $P(\text{blue first and red second}) = \frac{5}{22} \times \frac{8}{21} = \frac{20}{231} \approx 0.087$
The probability is about 8.7%.

Chapter 10

18. $P(\text{two odds} | \text{1st roll a 5}) = P(\text{2nd roll odd}) = \frac{3}{6} = \frac{1}{2}$

20. a. $P(A) = \frac{3}{6} = \frac{1}{2}$ b. $P(A \text{ and } B) = P(\text{even and 2}) = P(2) = \frac{1}{6}$

c. $P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$

27. $P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \Rightarrow P(A \text{ and } B) = P(B|A) \times P(A) = 0.27 \times 0.76 = 0.2052$

33. $P(\text{married} | \text{aged 20 to 29}) = \frac{4407 + 9913}{18,142 + 19,401} \approx 0.381 = 38.1\%$

39. Let A = purchased computer in electronics store and B = purchased software in electronics store.

Give $P(A) = 0.77$ and $P(A \text{ and } B) = 0.41$,

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.41}{0.77} \approx 0.532 = 53.2\%$$

Lesson 10.7

8. about 4 tosses

13. about 73%

19. about 8%

26. about 2 times