Chapter 9 **Parent Guide**
Conic Sections

Conic sections involve the study of the different ways a plane can intersect two cones connected at their vertex. Chapter 9 shows the geometric interpretation of some polynomial functions.

Page 562 shows the various ways a plane intersects with a double cone. Each can be represented on the coordinate plane, so this chapter will focus on graphing.

Points, lines, and intersecting lines have already been thoroughly addressed in previous chapters and in *Algebra 1*. The focus of Chapter 9 is on parabolas, circles, ellipses, and hyperbolas.

Conic sections are used in various ways, from the study of the universe to the manufacturing of lenses for cameras and flashlights.

Lesson 9.1 classifies the various conic sections and states the distance formula and midpoint formula. Lessons 9.2 through 9.5 analyze the graphs of parabolas, circles, ellipses, and hyperbolas, respectively. Lesson 9.6 discusses how to solve nonlinear systems of equations.

The following activity is an analysis of conic sections that you may enjoy doing with your child.

PROBLEM FOR DISCUSSION (See textbook page 562)

In the diagram shown on page 562, a slanted line is revolved all the way around a vertical line, the axis, in three-dimensional space. Because the two lines intersect, the result is a pair of cones that have one point in common. Although the diagram cannot show it, the two cones extend indefinitely both upward and downward, forming a double-napped cone.

The intersection of a double cone and a plane is called a conic section. What does the position of the plane have to do with the way it intersects with the double-napped cone?

1. Discuss why the plane does not have to be at a right angle to the vertical center of the cone in order to intersect at one point.

   Any plane that intersects the center outside of the cone will intersect at one point. It does not have to be at a right angle, but it does have to be at an angle that only intersects the center.

2. Discuss, in terms of the position of the intersecting plane, how a parabola and hyperbola are different.

   A parabola is formed when the intersecting plane is parallel to the revolving lines.

   A hyperbola is formed when the intersecting plane is parallel to the center axis.

3. Discuss, in terms of the position of the intersecting plane, how a circle and ellipse are different.

An ellipse is formed when the plane intersects at any angle. For a circle to be formed the plane has to intersect the cone horizontally. A circle is really just a special case of the ellipse.

4. Discuss why a point is categorized as an ellipse.

A point is the most special case of the ellipse. The plane intersects the cone at any angle at the center. This is just a specific example of an ellipse.

5. Discuss how the plane intersects the double-napped cone to create two intersecting lines.

When the plane intersects the cone at the center, cutting the cone in half, two intersecting lines are formed.

6. What is true about the plane that intersects the double-napped cone in one line?

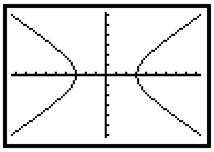
The plane lies on the revolving line of the cone.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 9.1

13. $4x^2 - 9y^2 = 36$

$$-9y^2 = 36 - 4x^2$$
$$9y^2 = 4x^2 - 36$$
$$3y = \pm\sqrt{4x^2 - 36}$$
$$3y = \pm 2\sqrt{x^2 - 9}$$
$$y = \pm \frac{2}{3}\sqrt{x^2 - 9}$$



hyperbola

26. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(-5 - 7)^2 + [-1 - (-2)]^2}$

$PQ = \sqrt{145} \approx 12.04$

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= M\left(\frac{7 + (-5)}{2}, \frac{-2 + (-1)}{2}\right)$

$= M\left(1, -\frac{3}{2}\right)$

39. center $= C\left(\frac{2 + (-3)}{2}, \frac{4 + 16}{2}\right) = C\left(-\frac{1}{2}, 10\right)$

radius $= CQ = \sqrt{\left[-3 - \left(-\frac{1}{2}\right)\right]^2 + (16 - 10)^2} = \sqrt{\frac{169}{4}} = \frac{13}{2}$

circumference $= 2\pi r = 2\pi\left(\frac{13}{2}\right) = 13\pi$

area $= \pi r^2 = \pi\left(\frac{13}{2}\right)^2 = \frac{169\pi}{4}$

49. $AB = \sqrt{(2 - 0)^2 + (4 - 0)^2} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(3 - 2)^2 + (7 - 4)^2} = \sqrt{10}$

$AC = \sqrt{(3 - 0)^2 + (7 - 0)^2} = \sqrt{58}$

Since $2\sqrt{5} + \sqrt{10} \neq \sqrt{58}$, the points are not collinear.

54. **a.** $M_{\overline{AC}} = \left(\frac{-4 + 4}{2}, \frac{4 + (-5)}{2}\right) = \left(0, -\frac{1}{2}\right)$

$M_{\overline{BD}} = \left(\frac{4 + (-4)}{2}, \frac{2 + (-3)}{2}\right) = \left(0, -\frac{1}{2}\right)$

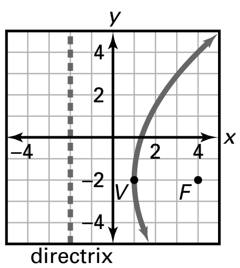
b. The diagonals intersect at their common midpoint.

Lesson 9.2

10. $p = 3, h = -2, k = 2$

$$x - h = \frac{1}{4p}(y - k)^2$$
$$x - (-2) = \frac{1}{4(3)}(y - 2)^2$$
$$x + 2 = \frac{1}{12}(y - 2)^2$$

22. Rewrite $x - 1 = \frac{1}{12}(y + 2)^2$ as
$x - 1 = \frac{1}{4(3)}[y - (-2)]^2$
Then $h = 1, k = -2$, and $p = 3$.
Since $p > 0$, the parabola opens to the right.
vertex: $(1, -2)$
focus: $(4, -2)$
directrix: $x = -2$



37. Since the focus is below the vertex, the parabola opens downward.
Since $0 + p = -5, p = -5$.
The equation is $y = \frac{1}{4p}x^2$, or $y = -\frac{1}{20}x^2$.

42. Since the vertex is to the right of the directrix, the parabola opens to the right.
Since $0 - p = -3, p = 3$.
The equation is $x = \frac{1}{4p}y^2$, or $x = \frac{1}{12}y^2$.

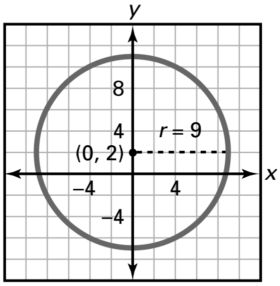
49.
$$4(x + 4) + (y - 2)^2 - 6(y - 2) = 9$$
$$4x + 16 + y^2 - 4y + 4 - 6y + 12 = 9$$
$$4x + 23 = -y^2 + 10y$$
$$4x + 23 - 5^2 = -(y^2 - 10y + 5^2)$$
$$4x - 2 = -(y - 5)^2$$
$$x - \frac{1}{2} = -\frac{1}{4}(y - 5)^2$$

Lesson 9.3

12. $[x - (-3)]^2 + (y - 4)^2 = \left(\frac{3}{2}\right)^2$
$(x + 3)^2 + (y - 4)^2 = \frac{9}{4}$, or 2.25

24. $(x - 0)^2 + (y - 12)^2 = 2^2$
$x^2 + (y - 12)^2 = 4$

37. $x^2 + (y - 2)^2 = 81$
$(x - 0) + (y - 2) = 9^2$
center (0, 2)
radius: 9



49. $x^2 + y^2 - 12x + 6y = 19$
$x^2 - 12x + y^2 + 6y = 19$
$(x^2 - 12x + 36) + (y^2 + 6y + 9) = 19 + 36 + 9$
$(x - 6)^2 + (y + 3)^2 = 64$
center: (6, −3)
radius: 8

64. circle; the equation can be written as $x^2 + (y - 2)^2 = 4$ which has the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 0$, $k = 2$, and $r = 2$.

78. The center of the original circle is at (1, −4). Translating 3 units to the left and 2 units down, the center becomes (1 + (−3), −4 + (−2)), or (−2, −6). Since the radius is unchanged, the standard equation of the resulting circle is $(x + 2)^2 + (y + 6)^2 = 16$.

Lesson 9.4

11. $\frac{x^2}{81} + \frac{y^2}{4} = 1$

The ellipse has a horizontal major axis with $a = 9$ and $b = 2$.

vertices: $(-9, 0)$ and $(9, 0)$

co-vertices: $(0, -2)$ and $(0, 2)$

18. $5x^2 + 20y^2 = 80$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

The ellipse has a horizontal major axis with $a = 4$ and $b = 2$.

Find c: $a^2 - b^2 = c^2$

$16 - 4 = c^2$

$c = \sqrt{12}$, or $2\sqrt{3}$

center: $(0, 0)$

fertices: $(-4, 0)$ and $(4, 0)$

co-vertices: $(0, -2)$ and $(0, 2)$

foci: $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$

23. The ellipse is centered at $(3, 3)$ with a horizontal major axis, $a = 4$, and $b = 3$.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-3)^2}{16} + \frac{(y-3)^2}{9} = 1$$

32. $\frac{(x-2)^2}{9} + \frac{(y-2)^2}{4} = 1$

The graph is an ellipse with horizontal major axis, $a = 3$, and $b = 2$.

$a^2 - b^2 = c^2$

$9 - 4 = c^2$

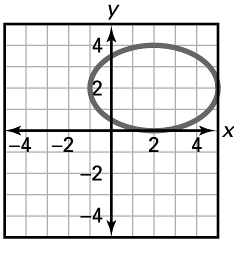
$\sqrt{5} = c$

center: $(2, 2)$

vertices: $(-1, 2)$ and $(5, 2)$

co-vertices: $(2, 0)$ and $(2, 4)$

foci: $(2 - \sqrt{5}, 2)$ and $(2 + \sqrt{5}, 2)$



54.

$$\begin{aligned} 4x^2 + 9y^2 - 16x + 18y &= 11 \\ 4x^2 - 16x + 9y^2 + 18y &= 11 \\ 4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) &= 11 + 16 + 9 \\ 4(x - 2)^2 + 9(y + 1)^2 &= 36 \\ \frac{(x - 2)^2}{9} + \frac{(y + 1)^2}{4} &= 1 \end{aligned}$$

The graph is an ellipse with horizontal major axis, $a = 3$, and $b = 2$.

Find c:

$$\begin{aligned} a^2 - b^2 &= c^2 \\ 9 - 4 &= c^2 \\ \sqrt{5} &= c \end{aligned}$$

center: $(2, -1)$

vertices: $(-1, -1)$ and $(5, -1)$

co-vertices: $(2, -3)$ and $(2, 1)$

foci: $(2 - \sqrt{5}, -1)$ and $(2 + \sqrt{5}, -1)$

62. a. We assume that the major axis is horizontal.

Since $2a = 774{,}000$ and $2b = 773{,}000$, we have $a = 387{,}000$ and $b = 386{,}500$.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{(387{,}000)^2} + \frac{y^2}{(386{,}500)^2} = 1$$

b. Find c:

$$\begin{aligned} a^2 - b^2 &= c^2 \\ (387{,}000)^2 - (386{,}500)^2 &= c^2 \\ 386{,}075{,}000 &= c^2 \\ c &= \sqrt{386{,}750{,}000} \approx 19{,}666 \end{aligned}$$

The closest approach corresponds to $a - c \approx 387{,}000 - 19{,}666 = 367{,}334$ kilometers.

c. The farthest point corresponds to $a + c \approx 387{,}000 + 19{,}666 = 406{,}666$ kilometers.

d. $E = \frac{c}{a} \approx \frac{19{,}666}{387{,}000} \approx 0.051$

Lesson 9.5

10. The center is at $(0, 0)$, the transverse axis is vertical, $a = 1$, and $b = 3$.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{1} - \frac{x^2}{9} = 1$$

21. $4x^2 - 25y^2 = 100$

$\frac{x^2}{25} - \frac{y^2}{4} = 1$

The graph is a hyperbola with a horizontal transverse axis, $a = 5$, and $b = 2$.

$a^2 + b^2 = c^2$

$25 + 4 = c^2$

$\sqrt{29} = c$

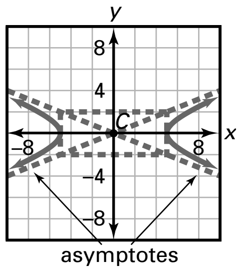
center: $(0, 0)$

vertices: $(-5, 0)$ and $(5, 0)$

co-vertices: $(0, -2)$ and $(0, 2)$

foci: $(-\sqrt{29}, 0)$ and $(\sqrt{29}, 0)$

asymptotes: $y = -\frac{2}{5}x$ and $y = \frac{2}{5}x$



30. The hyperbola has a vertical transverse axis, $b = 1$, and $c = 2$.

Find a: $a^2 + b^2 = c^2$

$a^2 + 1 = 4$

$a = \sqrt{3}$

vertices: $(0, \sqrt{3})$ and $(0, -\sqrt{3})$

$\frac{y^2}{3} - \frac{x^2}{1} = 1$

38. $16x^2 - 9y^2 + 64x = 89 - 18y$

$16x^2 + 64x - 9y^2 + 18y = 89$

$16(x^2 + 4x + 4) - 9(y^2 - 2y + 1) = 89 + 64 - 9$

$16(x + 2)^2 - 9(y - 1)^2 = 144$

$\frac{(x + 2)^2}{9} - \frac{(y - 1)^2}{16} = 1$

The graph is a hyperbola with a horizontal transverse axis, $a = 3$, and $b = 4$.

$a^2 + b^2 = c^2$

$9 + 16 = c^2$

$5 = c$

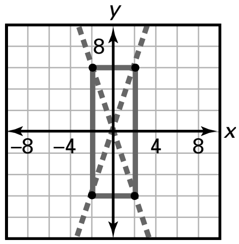
center: $(-2, 1)$

vertices: $(-5, 1)$ and $(1, 1)$

co-vertices: $(-2, -3)$ and $(-2, 5)$

foci: $(-7, 1)$ and $(3, 1)$

44.



The rectangle and asymptotes are shown.
The center is at (0, 0).
If the transverse axis is horizontal, then $a = 2$ and $b = 6$, so the standard equation is $\frac{x^2}{4} - \frac{y^2}{36} = 1$.
If the transverse axis is vertical, then $a = 6$ and $b = 2$, so the standard equation is $\frac{y^2}{36} - \frac{x^2}{4} = 1$.

49. Original hyperbola:

$$9x^2 - 4y^2 + 54x + 8y + 41 = 0$$
$$9x^2 + 54x - 4y^2 + 8y = -41$$
$$9(x^2 + 6x) - 4(y^2 - 2y) = -41$$
$$9(x^2 + 6x + 9) - 4(y^2 - 2y + 1) = -41 + 81 - 4$$
$$9(x + 3)^2 - 4(y - 1)^2 = 36$$
$$\frac{(x + 3)^2}{4} - \frac{(y - 1)^2}{9} = 1$$

Translated hyperbola:

$$\frac{[(x + 6) + 3]^2}{4} - \frac{[(y - 2) - 1]^2}{9} = 1$$
$$\frac{(x + 9)^2}{4} - \frac{(y - 3)^2}{9} = 1$$

Lesson 9.6

16. $\begin{cases} y = x^2 \\ x^2 - y^2 = 4 \end{cases}$

Substitute y for x^2 in $x^2 - y^2 = 4$.

$y - y^2 = 4$

$0 = y^2 - y + 4$

$y = \frac{1 \pm \sqrt{1^2 - 4(1)(4)}}{2(1)}$

$y = \frac{1 \pm \sqrt{-15}}{2}$

$y = \frac{1}{2} - \frac{\sqrt{15}}{2}i$ or $y = \frac{1}{2} + \frac{\sqrt{15}}{2}i$

There are no real solutions.

27. $\begin{cases} x^2 - y^2 = 36 \\ 4y^2 - 9x^2 = 36 \end{cases}$

$$\rightarrow \begin{cases} 4x^2 - 4y^2 = 144 \\ -9x^2 + 4y^2 = 36 \end{cases}$$
$$-5x^2 = 180$$
$$x^2 = -36$$
$$x = \pm 6i$$

There are no real solutions.

37. $\begin{cases} x^2 - 4y^2 = 20 \\ y^2 = 4 \end{cases}$

Solve $y^2 = 4$ for y.

$$y^2 = 4$$
$$y = \pm 2$$

Substitute 4 for y^2 in $x^2 - 4y^2 = 20$

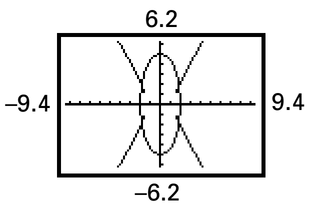
$$x^2 - 4y^2 = 20$$
$$x^2 - 4(4) = 20$$
$$x^2 - 16 = 20$$
$$x^2 = 36$$
$$x = \pm 6$$

There are four solutions: $(\pm 6, \pm 2)$

43. Solve each equation for y.

$$4y^2 + 25x^2 = 100 \qquad 5x^2 - 2y^2 = 10$$
$$4y^2 = 100 - 25x^2 \qquad 5x^2 - 10 = 2y^2$$
$$2y = \pm 5\sqrt{4 - x^2} \qquad \tfrac{5}{2}x^2 - 5 = y^2$$
$$y = \pm \tfrac{5}{2}\sqrt{4 - x^2} \qquad y = \pm\sqrt{\tfrac{5}{2}x^2 - 5}$$

There are four approximate solutions: $(\pm 1.85, \pm 1.89)$



51. The equation can be written as $4x^2 - 9y^2 + 8x + 36y - 68 = 0$, so $A = 4$ and $C = -9$. Since $4(-9) < 0$, the graph is a hyperbola.

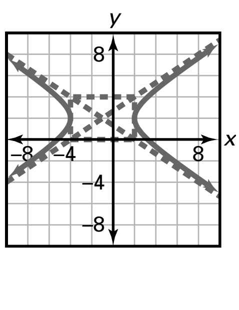
$$4x^2 + 8x - 9y^2 + 36y - 68 = 0$$
$$4x^2 + 8x - 9y^2 + 36y = 68$$
$$4(x^2 + 2x + 1) - 9(y^2 - 4y + 4) = 68 + 4 - 36$$
$$4(x + 1)^2 - 9(y - 2)^2 = 36$$
$$\frac{(x + 1)^2}{9} - \frac{(y - 2)^2}{4} = 1$$

The graph is a hyperbola with a horizontal transverse axis, $a = 3$, and $b = 2$.

center: $(-1, 2)$

vertices: $(-4, 2)$ and $(2, 2)$

co-vertices: $(-1, 0)$ and $(-1, 4)$



62. $\begin{cases} \dfrac{(x-1)^2}{9} + \dfrac{y^2}{4} = 1 \\ \dfrac{(x-1)^2}{9} + y^2 = 1 \end{cases}$

$$\rightarrow -\frac{(x-1)^2}{9} - \frac{y^2}{4} = -1$$
$$\frac{(x-1)^2}{9} + y^2 = 1$$
$$\frac{3}{4}y^2 = 0$$
$$y^2 = 0$$
$$y = 0$$

Substitute 0 for y^2 in $\frac{(x-1)^2}{9} + y^2 = 1$.

$$\frac{(x-1)^2}{9} + y^2 = 1$$
$$\frac{(x-1)^2}{9} + 0 = 1$$
$$(x-1)^2 = 9$$
$$x - 1 = \pm 3$$
$$x = 1 \pm 3$$

$x = -2$ or $x = 4$

The solutions are $(-2, 0)$ and $(4, 0)$.