Chapter 8 — Parent Guide
Rational Functions and Radical Functions

Rational functions consist of one polynomial function divided by another. Radical functions contain radical expressions in them.

These types of functions occur in various fields, including physics, chemistry, engineering, and economics. They also occur in advanced mathematics.

Chapter 8 first develops an understanding of rational functions, and develops radical function concepts in the latter part of the chapter. It will apply concepts discussed in Chapter 7.

Lesson 8.1 begins the study of rational functions with a discussion on inverse, joint, and combined variation. Lesson 8.2 identifies, evaluates, and graphs rational functions.

Lesson 8.3 involves multiplying, dividing and simplifying rational expressions. In Lesson 8.4, your child will add and subtract rational expressions. Lesson 8.5 concludes the study of rational functions with solving rational equations and inequalities. Lesson 8.6 opens the discussion of radical expressions and functions by analyzing their graphs and evaluating radical expressions. Lesson 8.7 involves computations with radical expressions as well as simplifying them. In Lesson 8.8, your child will solve radical equations and radical inequalities.

Your child will need to master computations involving rational functions and rational expressions to be successful in the study of conic sections in Chapter 10. You can help your son or daughter understand rational equations by doing the following activity together.

PROBLEM FOR DISCUSSION (See textbook page 512)

Rachel finished a triathlon involving swimming, bicycling, and running in 2.5 hours. Rachel's bicycling speed was about 6 times her swimming speed, and her running speed was about 5 miles per hour greater than her swimming speed. Find the speeds at which Rachel competed. Use s to represent Rachel's swimming speed.

1. Discuss how to use the distance formula to write an expression relating the time for each event.

The distance formula is simply distance equals the product of the rate and time, $d = rt$. The problem tells you that the total time for Rachel's race is $2\frac{1}{2}$ hours. Since this is the total time, you need to find the time of each part of the race. So, first solve $d = rt$ for t.

$d = rt$ Divide both sides by r.

$\frac{d}{r} = \frac{rt}{r}$

$\frac{d}{r} = t$

Use this formula to write an expression for each portion of the race.

Swimming:
On page 512 the chart tells you that the distance for the swimming portion of the race is 0.5 miles. The problem tells you to let s be the time it took for Rachel to swim the race. Using $\frac{d}{r} = t$, substitute 0.5 miles for d and s for r.

The expression is $\frac{0.5}{s} = t$.

Bicycling:
On page 512 the chart tells you that the distance for the bicycling portion of the race is 25 miles. It also tells you that her speed during the bicycling portion is 6 times that of her swimming speed. Since s is her swimming speed, $6s$ is her bicycling speed. Using $\frac{d}{r} = t$, substitute 25 miles for d and $6s$ for r.

The expression is $\frac{25}{6s} = t$.

Running:
On page 512 the chart tells you that the distance for the running portion of the race is 6 miles. It also tells you that her speed during the running portion is 5 miles per hour more than that of her swimming speed. Since s is her swimming speed, $s + 5$ is her bicycling speed. Using $\frac{d}{r} = t$, substitute 6 miles for d and $s + 5$ for r.

The expression is $\frac{6}{s+5} = t$.

2. Discuss why each should be a function of time and not speed.

Each expression is a function of time because the problem is based on the fact that the race was completed in a fixed amount of time, $2\frac{1}{2}$ hours.

3. Discuss why each function should be added.

The total time includes the time it took for her to complete the swimming portion, the bicycling portion, and the running portion.
In other words,
swimming time + bicycling time + running time = total time.
Each function should be added together and set equal to the total time.

4. Look at Part 2 of Example 1. Compare adding rational expressions to adding fractions.

Adding rational expression is the same as adding fractions. In order to combine rational expressions, a common denominator is needed.

5. Discuss why a variable appears in the denominator of the function at the end of Part 2 of Example 1?

A variable appears in the denominator of the final expression because the common denominator for the expressions $\frac{0.5}{s}$, $\frac{25}{6s}$, and $\frac{6}{s+5}$, involves the variable s. So, the total time is only true for values of s that do not make each of the denominators 0.

6. Discuss how a rational function differs from a polynomial function.

A rational function has variables in the denominator. Polynomial functions do not have variables in the denominator.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Lesson 8.1

14.
$$xy = k$$
$$(5)(10) = k$$
$$k = 50$$

An equation for the relationship is $y = \frac{50}{x}$.

x	y
2.5	20
3	16.667
3.5	14.286

25.
$$\frac{y}{xz} = k$$
$$\frac{120}{(8)(20)} = k$$
$$k = \frac{3}{4}$$

An equation for the relationship is $y = \frac{3}{4}xz$.

$$y = \frac{3}{4}xz = \frac{3}{4}(54)(7) = 283.5$$

31.
$$\frac{wz}{xy} = k$$
$$\frac{(15)(36)}{(9)(10)} = k$$
$$k = 6$$

An equation for the relationship is $z = \frac{6xy}{w}$.

$$z = \frac{6xy}{w} = \frac{6(10)(18)}{5} = 216$$

38. $x_1y_1 = x_2y_2$
$$3y = (18)(6)$$
$$y = 36$$

44. Let the lifting force be F, the surface area of the wings be A, and the air speed be s. Then

$$F = kAs^2$$
$$k = \frac{F}{As^2}$$

If the plane has a cruising speed of 250 miles per hour, then

$$k = \frac{F}{A \cdot 250^2}$$

To triple the lifting force, by doubling the surface area, we find the necessary cruising speed as follows:

$$3F = k(2A)s^2$$
$$k = \frac{3F}{2As^2}$$
$$\frac{F}{A \cdot 250^2} = \frac{3F}{2As^2}$$
$$s^2 = \frac{3}{2}250^2$$
$$s = \sqrt{\frac{3}{2}}\,250$$
$$s \approx 306$$

The cruising speed will be about 306 mph.

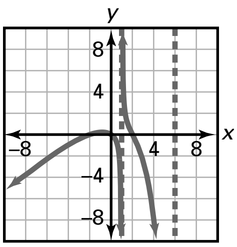
Lesson 8.2

12. $g(x) = \frac{x+2}{2x}$ is a rational function. The domain is all real numbers except $x = 0$.

22. $b(x) = \frac{x^2 - 2x + 1}{x^2 + x - 2} = \frac{(x-1)^2}{(x+2)(x-1)} = \frac{x-1}{x+2}$

There is a hole in the graph when $x = 1$.
There is a vertical asymptote when $x = -2$.
There is a horizontal asymptote when $y = 1$.

31. $m(x) = \frac{x(x^2 - 4)}{x^2 - 7x + 6} = \frac{x(x+2)(x-2)}{(x-1)(x-6)}$



The domain is all real numbers except $x = 1$ and $x = 6$. There are vertical asymptotes at $x = 1$ and $x = 6$. There are no horizontal asymptotes and no holes in the graph.

37. Answers may vary. If $x = \pm 1$ are vertical asymptotes, $x - 1$, and $x + 1$ appear in the denominator. If $y = 1.5$ is a horizontal asymptote, numerator may contain $1.5x^2$. If there is a hole at $x = 0$, x is a common factor in the numerator and denominator.

$$f(x) = \frac{x \cdot 1.5x^2}{x(x-1)(x+1)} = 1.5\frac{x^3}{x^3 - x} = \frac{3}{2} \cdot \frac{x^2}{x^3 - x}$$

$$\text{So } f(x) = \frac{3x^3}{2x^3 - 2x}$$

43. **a.** $R(x) = \frac{2\pi x}{\pi x^2}$

$R(x) = \frac{2}{x}$

b. (Refer to 42 **b** for further explanation). C, A, and x all need to be positive, so $x > 0$ defines the domain for C, A, and R.

Lesson 8.3

11. $\frac{15}{x^2} \cdot \frac{x^5}{12} \cdot \frac{4}{x} = 5x^2$

20. $$\frac{4x^2 + 20x}{9 + 6x + x^2} \div \frac{x + 5}{x^2 - 9} = \frac{4x^2 + 20x}{9 + 6x + x^2} \cdot \frac{x^2 - 9}{x + 5}$$
$$= \frac{4x(x + 5)}{(3 + x)^2} \cdot \frac{(x + 3)(x - 3)}{x + 5}$$
$$= \frac{4x(x - 3)}{(x + 3)} = \frac{4x^2 - 12x}{x + 3}$$

31. $$\frac{\frac{x^2 - 9x + 14}{x^2 - 6x + 5}}{\frac{x^2 - 8x + 7}{x^2 - 7x + 10}} = \frac{x^2 - 9x + 14}{x^2 - 6x + 5} \cdot \frac{x^2 - 7x + 10}{x^2 - 8x + 7}$$
$$= \frac{(x - 2)(x - 7)}{(x - 1)(x - 5)} \cdot \frac{(x - 2)(x - 5)}{(x - 1)(x - 7)}$$
$$= \frac{(x - 2)^2}{(x - 1)^2} = \frac{x^2 - 4x + 4}{x^2 - 2x + 1}$$

35. $$\frac{2x + 3}{x - 1} \div \frac{\frac{x}{x - 1}}{\frac{3x}{2x + 3}} = \frac{2x + 3}{x - 1} \cdot \frac{\frac{3x}{2x + 3}}{\frac{x}{x - 1}}$$
$$= \frac{2x + 3}{x - 1} \cdot \frac{3x}{2x + 3} \cdot \frac{x - 1}{x}$$
$$= 3$$

46. a. Average revenue per sandwich $= \frac{3.50 + 4.50 + 3.75 + 3.75 + 3.25}{5}$
$$= 3.75$$

The average revenue per sandwich is $3.75.

b. $C(x) = 1.69x + 1200$

c. $R(x) = \frac{3.75}{\frac{1.69x + 1200}{x}}$

$R(x) = \frac{3.75x}{1.69x + 1200}$

Lesson 8.4

13. $$\frac{x + 7}{3} - \frac{4x + 1}{9} = \frac{x + 7}{3} \cdot \frac{(3)}{(3)} - \frac{4x + 1}{9}$$
$$= \frac{3(x + 7) - (4x + 1)}{9}$$
$$= \frac{3x + 21 - 4x - 1}{9}$$
$$= \frac{-x + 20}{9}$$

26. $$\frac{\frac{1}{3x + 1}}{2} = \frac{1}{3x + 1} \cdot \frac{1}{2}$$
$$= \frac{1}{2(3x + 1)} = \frac{1}{6x + 2}$$

38. $(a-b)^{-1} - (a-b)^{-1} = \frac{1}{a-b} - \frac{1}{a+b}$

$$= \frac{(a+b)}{(a-b)(a+b)} - \frac{(a-b)}{(a+b)(a-b)}$$
$$= \frac{a+b-(a-b)}{(a-b)(a+b)}$$
$$= \frac{2b}{(a-b)(a+b)} = \frac{2b}{a^2-b^2}$$

48. Let the common distance be d, and

t_1 be the time to travel from A to B
t_2 be the time to travel from B to A
t_3 be the time to travel from B to C
t_4 be the time to travel from C to B
t_5 be the time to travel from C to D
t_6 be the time to travel from D to C

From A to C and back to A, the average speed will be

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$
$$= \frac{d+d+d+d}{t_1+t_3+t_4+t_2}$$
$$= \frac{4d}{\frac{d}{35}+\frac{d}{50}+\frac{d}{42}+\frac{d}{30}}$$
$$= \frac{4d}{\frac{111d}{1050}}$$
$$= \frac{4200}{111}$$
$$\approx 37.8$$

Justine's average speed was about 37.8 mph.

Lesson 8.5

11. $\frac{4}{n+4} = 1$

$$4 = n + 4$$
$$n = 0$$

check: 0 is not an excluded value

$n = 0$

23.

$$\frac{2x+3}{x-1} - \frac{2x-3}{x+1} = \frac{10}{x^2-1}$$
$$\frac{2x+3}{x-1}(x+1)(x-1) - \frac{2x-3}{x+1}(x+1)(x-1) = \frac{10}{(x+1)(x-1)}(x+1)(x-1)$$
$$(2x+3)(x+1) - (2x-3)(x-1) = 10$$
$$2x^2+5x+3-(2x^2-5x+3) = 10$$
$$10x = 10$$
$$x = 1$$

check: 1 is an excluded value
No solution.

34. $3 < \frac{3x+4}{2x+1}$

Case 1: Assume

$2x + 1 > 0 \rightarrow x > -\frac{1}{2}$

$$6x + 3 < 3x + 4$$
$$3x < 1$$
$$x < \frac{1}{3}$$

Thus $-\frac{1}{2} < x < \frac{1}{3}$.

Case 2: Assume

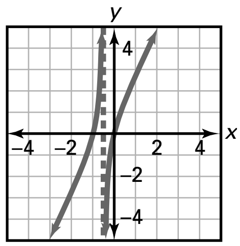
$2x + 1 < 0 \rightarrow x < -\frac{1}{2}$

$$6x + 3 > 3x + 4$$
$$3x > 1$$
$$x > \frac{1}{3}$$

No solution.

42. $$2x + 1 \geq \frac{1}{2x+1}$$
$$2x + 1 - \frac{1}{2x+1} \geq 0$$

Graph $y = 2x + 1 - \frac{1}{2x+1}$ and find the values of x for which $y \geq 0$



$-1 \leq x < -\frac{1}{2}$ or $x \geq 0$

56. a. Let the speed Michael swims be s. Then we have the table

	distance (mi)	speed (mph)
swimming	0.6	s
bicycling	15	$9s$
running	8	$s + 6$

Swimming time

$$d_s = rt_s$$
$$0.6 = (s)t_s$$
$$t_s = \frac{0.6}{s}$$

bicycling time

$$d_b = rt_b$$
$$15 = (9s)t_b$$
$$t_b = \frac{15}{9s}$$

running time

$$d_r = rt_r$$
$$8 = (s + 6)t_r$$
$$t_r = \frac{8}{s+6}$$

Let $T(s)$ be the total time.

Then $T(s) = \frac{0.6}{s} + \frac{15}{9s} + \frac{8}{s+6}$

b.
$$T(s) = 1.5$$
$$\frac{0.6}{s} + \frac{15}{9s} + \frac{8}{s+6} = 1.5$$
$$\frac{0.6}{s}[9s(s+6)] + \frac{15}{9s}[9s(s+6)] + \frac{8}{s+6}[9s(s+6)] = 1.5[9s(s+6)]$$
$$5.4(s+6) + 15(s+6) + 72s = 13.5s(s+6)$$
$$92.4s + 122.4 = 13.5s^2 + 81s$$
$$13.5s^2 - 11.4s - 122.4 = 0$$

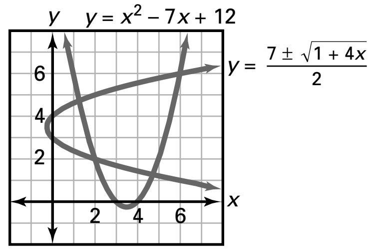
Solve by graphing and using the calculator's trace feature or use the quadratic formula: $s \approx 3.5$. To complete his workout in 1.5 hours, Michael must swim at about 3.5 mph, he must bicycle at about 31.5 mph, and he must run at about 9.5 mph.

Lesson 8.6

16. $3 - 2(x - 4) \geq 0$

$$-2(x-4) \geq -3$$
$$x - 4 \leq \frac{3}{2}$$
$$x \leq 4 + \frac{3}{2}$$
$$x \leq \frac{11}{2}$$

32.



$y = x^2 - 7x + 12 \rightarrow$

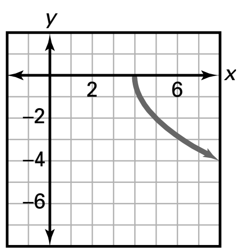
$$x = y^2 - 7x + 12$$
$$y^2 - 7x + 12 - x = 0$$
$$y = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12 - x)}}{2(1)}$$
$$y = \frac{7 \pm \sqrt{1 + 4x}}{2}$$

41. $\frac{1}{8}(\sqrt[3]{-8^3})^2 = \frac{1}{8}(\sqrt[3]{-512})^2$

$$= \frac{1}{8}(-8)^{22}$$
$$= 8$$

58. $s(x) = -2\sqrt{x - 4}$

A horizontal translation 4 units right then a vertical stretch by a factor of 2 and a reflection across the x-axis



65. **a.** $E \approx 1000(100 - T) + 580(100 - T)^2$
Let $100 - T = x$ and re-write in terms of x.
$E \approx 1000x + 580x^2$

$$580x^2 + 1000x - E \approx 0$$
$$x \approx \frac{-1000 \pm \sqrt{1000^2 - 4(580)(-E)}}{2(580)}$$
$$x \approx \frac{-1000 \pm \sqrt{1{,}000{,}000 + 2320E}}{1160}$$
$$x \approx \frac{-1000 \pm 4\sqrt{5(12{,}500 + 29E)}}{1160}$$
$$x \approx \frac{-250 \pm \sqrt{5(12{,}500 + 29E)}}{290}$$

Resubstitute $x = 100 - T$
$$100 - T \approx \frac{-250 \pm \sqrt{5(12{,}500 + 29E)}}{290}$$
$$-T \approx -100 + \left(\frac{-250 \pm \sqrt{5(12{,}500 + 29E)}}{290}\right)$$
$$T \approx 100 + \frac{250 \pm \sqrt{5(12{,}500 + 29E)}}{290}$$
$$T \approx \frac{29{,}250 \pm \sqrt{5(12{,}500 + 29E)}}{290}$$

b. For $E = 1600$,
$$T \approx \frac{29{,}250 \pm \sqrt{5(12{,}500 + 29 \cdot 1600)}}{290}$$
$$T \approx 99 \text{ or } 103$$

T must be less than or equal to 100, because the original equation is defined for $E \geq 0$, so $100 - T \geq 0$, or $T \leq 100$.
Water will boil at about 99° Celsius.

Lesson 8.7

19. $\sqrt{18x^3} = \sqrt{3^2x^2 \cdot 2x}$
$= 3|x|\sqrt{2x}$

24. $\sqrt[3]{24x^5y^3z^9} = \sqrt[3]{2^3x^3y^3z^9 \cdot 3x^2}$
$= 2xyz^3\sqrt[3]{3x^2}$

33. $(24rs)^{\frac{1}{2}} \cdot \sqrt{6r^3s^4}\sqrt{rs^2} = \sqrt{24rs} \cdot \sqrt{6r^3s^4} \cdot \sqrt{rs^2}$
$= \sqrt{24rs \cdot 6r^3s^4 \cdot rs^2}$
$= \sqrt{12^2r^4s^6 \cdot rs}$
$= 12r^2s^3\sqrt{rs}$

50. $(5 + \sqrt{125}) + (-10 + 10\sqrt{5}) = (5 + \sqrt{5^2 \cdot 5}) + (-10 + 10\sqrt{5})$
$= (5 + 5\sqrt{5}) + (-10 + 10\sqrt{5})$
$= 5 - 10 + 5\sqrt{5} + 10\sqrt{5}$
$= -5 + 15\sqrt{5}$

61. $(4 - 2\sqrt{27})(1 + \sqrt{75}) = (4 - 2\sqrt{3^2 \cdot 3})(1 + \sqrt{5^2 \cdot 3})$
$= (4 - 6\sqrt{3})(1 + 5\sqrt{3})$
$= (4)(1) + (4)(5\sqrt{3}) + (-6\sqrt{3})(1) + (-6\sqrt{3})(5\sqrt{3})$
$= 4 + 20\sqrt{3} - 6\sqrt{3} - 90$
$= -86 + 14\sqrt{3}$

80. $\frac{\sqrt{3}}{\sqrt{27}} = \sqrt{\frac{3}{27}}$
$= \sqrt{\frac{1}{9}}$
$= \sqrt{\frac{1}{3^2}}$
$= \frac{1}{3}$

Lesson 8.8

16. $2\sqrt{x} = 3\sqrt{x - 2}$
$(2\sqrt{x})^2 = (3\sqrt{x - 2})^2$
$4x = 9(x - 2)$
$18 = 5x$
$x = \frac{18}{5}$

check: $2\sqrt{\frac{18}{5}} \stackrel{?}{=} 3\sqrt{\frac{18}{5} - 2}$
$2\sqrt{\frac{9 \cdot 2}{5}} \stackrel{?}{=} 3\sqrt{\frac{8}{5}}$
$2 \cdot 3\sqrt{\frac{2}{5}} \stackrel{?}{=} 3 \cdot 2\sqrt{\frac{2}{5}}$
$6\sqrt{\frac{2}{5}} \stackrel{?}{=} 6\sqrt{\frac{2}{5}}$ **True**

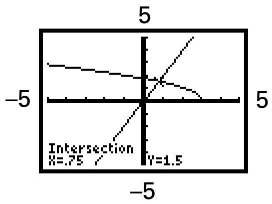
The solution is $x = \frac{18}{5}$.

31. $1 > 3\sqrt{3x - 1}$ $\quad$ $3x - 1 \geq 0$
$1^2 > (3\sqrt{3x - 1})^2$ $\quad$ $3x \geq 1$
$1 > 9(3x - 1)$ $\quad$ $x \geq \frac{1}{3}$
$10 > 27x$
$x < \frac{10}{27}$

check: by graphing

The solution is $\frac{1}{3} \leq x < \frac{10}{27}$.

46. $2x > \sqrt{3 - x}$

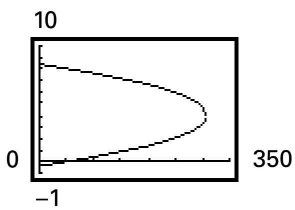


$0.8 < x \leq 3$

57.
$$2\sqrt[4]{3x} = \sqrt[4]{3x + 15}$$
$$16(3x) = 3x + 15$$
$$48x - 3x = 15$$
$$45x = 15$$
$$x = \frac{1}{3}$$
sometimes

62. a.
$$h = -16t^2 + 128t + 50$$
$$-16t^2 + 128t + 50 - h = 0$$
$$t = \frac{-128 \pm \sqrt{128^2 - 4(-16)(50 - h)}}{2(-16)}$$
$$= \frac{-128 \pm \sqrt{19584 - 64h}}{-32}$$
$$= 4 \pm \frac{\sqrt{306 - h}}{4}$$
$$t = 4 + \frac{\sqrt{306 - h}}{4} \text{ or } t = 4 - \frac{\sqrt{306 - h}}{4}$$

b.



c. $t = 4 - \frac{\sqrt{306 - h}}{4}$ or $t = 4 + \frac{\sqrt{306 - h}}{4}$

$t \approx 2.5$ or $t \approx 5.5$

The projectile will be at 270 feet after about 2.5 seconds and after about 5.5 seconds.