

## Chapter 7 Parent Guide Polynomial Functions

**Polynomial functions are classified** according to their degree. Your child has already studied linear and quadratic functions. These are two types of polynomial functions. To continue studying in mathematics or science, your child will need to learn more about all types of polynomial functions.

Polynomial functions have applications in all the sciences and financial analyses. People who have an understanding of a wide variety of functions and how to use and represent them will broaden their career choices and increase their chances for quick advancement in their chosen career.

Chapter 7 serves as a refresher of what was learned in Algebra 1, yet will also introduce new concepts, such as polynomial long division and synthetic division.

This chapter and Chapter 8 will serve as a backdrop for conic sections studied in Chapter 9. Your child will need a solid understanding of polynomial functions before approaching Chapters 8 and 9.

Lesson 7.1 classifies, evaluates, adds, and subtracts polynomials. Lesson 7.2 analyzes and graphs polynomial functions. Lesson 7.3 multiplies polynomials and uses long and synthetic division to divide polynomials. Lesson 7.4 solves polynomial equations by graphing and factoring. Lesson 7.5 finds zeros of a polynomial function, including complex zeros.

You may enjoy doing the following activity with your child. It involves some geometric and algebraic principles. Your daughter or son may recall working a similar problem in *Algebra 1* or *Geometry*.

### PROBLEM FOR DISCUSSION (See textbook page 440)

Making an open-top box out of a single rectangular sheet involves cutting and folding square flaps at each of the corners. These flaps are then pasted to the adjacent side to provide reinforcement for the corners.

The dimensions of the rectangular sheet and the square flaps determine the volume of the resulting box. Write a polynomial for the outside surface area and volume of the box.

1. Write three algebraic expressions to represent the length, width, and height of the box. Discuss why the length and width contain the monomial  $2x$  instead of  $x$ , like the height.

The length of the original piece of cardboard is 16 inches. After the two pieces are cut off of the cardboard, the length will be shorter than 16 inches. You don't know how much shorter because you don't know exactly how much was cut off. All you know is that  $x$  inches was cut off of both sides. Therefore, the length of the box is  $16 - 2x$  inches. You subtract  $2x$  instead of  $1x$  because  $1x$  is cut off of both sides.

The width of the original piece of cardboard is 12 inches. After the two pieces are cut off of the cardboard, the length will be shorter than 12 inches. You don't know how much shorter because you don't know exactly how much was cut off. All you know is that  $x$  inches was cut off of both sides. Therefore, the length of the box is  $12 - 2x$  inches. You subtract  $2x$  instead

of  $1x$  because  $1x$  is cut off of both sides.

The height of the box is the same as the length of the squares cut off.  
Therefore, the height of the box is  $x$ .

2. Discuss how to use the expressions in Exercise 1 to write an equation for the volume of the box. What is the degree of the equation?

To write an equation, remember how to find the volume of a box.

$$V = l \times w \times h$$

$$V = (16 - 2x)(12 - 2x)(x)$$

$$V = (192 - 56x + 4x^2)(x)$$

$$V = 192x - 56x^2 + 4x^3$$

Since the largest exponent is 3, the degree of this equation is 3.

3. Write an algebraic expression to represent the area of the exterior face of the box. How many different expressions do you need to write?

There are 5 faces of this box. The area of opposite sides is the same.  
Therefore, there will be three different expressions for this box.

The area for the shorter sides of the box:

Remember,  $A = lw$ .

The length of the side is  $12 - 2x$  and the width is  $x$ .

$$A = (12 - 2x)(x)$$

$$A = 12x - 2x^2$$

The area for the longer sides of the box:

$A = lw$ .

The length of the side is  $16 - 2x$  and the width is  $x$ .

$$A = (16 - 2x)(x)$$

$$A = 16x - 2x^2$$

The area for the bottom of the box:

$A = lw$ .

The length of the side is  $16 - 2x$  and the width is  $12 - 2x$ .

$$A = (16 - 2x)(12 - 2x)$$

$$A = 192 - 56x + 4x^2$$

4. Discuss how to use the expressions in Exercise 2 to write an equation for the surface area of the box. What is the degree of the equation?

To write an equation for the surface area of the box, the area of each of the 5 sides need to be added together.

Use the expressions given above to write the sum for the surface area.

$$SA = \text{short side} + \text{short side} + \text{long side} + \text{long side} + \text{bottom}$$

$$SA = 12x - 2x^2 + 12x - 2x^2 + 16x - 2x^2 + 16x - 2x^2 + 192 - 56x + 4x^2$$

$$SA = 2(12x - 2x^2) + 2(16x - 2x^2) + 192 - 56x + 4x^2$$

$$SA = 24x - 4x^2 + 32x - 4x^2 + 192 - 56x + 4x^2$$

$$SA = 192 - 4x^2$$

5. Discuss why each of these equations are polynomial functions.

Each of these equations are polynomial functions because they all have polynomial expressions in them.

The following are complete worked out solutions to selected exercises in the student textbook.

These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

### Lesson 7.1

$$14. \quad 9.1x^2 + 5.4x^5 + 3.3x^2 + 2.1 = 5.4x^5 + (9.1x^2 + 3.3x^2) + 2.1 \\ = 5.4x^5 + 12.4x^2 + 2.1$$

25. Yes,  $\frac{5}{7}x^6 + \frac{2}{3}x^4 + 5$  is a polynomial.

The greatest exponent of  $x$  is 6 and the polynomial has 3 terms so it is a trinomial of degree 6.

$$33. \quad 3x^3 + x^2 + 2x + 4 = 3(5)^3 + 5^2 + 2(5) + 4 \\ = 375 + 25 + 10 + 4 \\ = 414$$

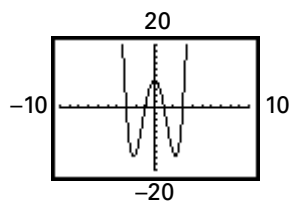
$$40. \quad (x^5 + x^3 + x) + (x^4 + x^2 + 1) = x^5 + x^4 + x^3 + x^2 + x + 1$$

This is a quintic polynomial.

$$48. \quad \left(\frac{2}{7}x^2 + \frac{1}{7}x + \frac{3}{7}\right) - \left(\frac{4}{7}x^3 + \frac{6}{7}x^2 + \frac{2}{7}\right) = -\frac{4}{7}x^3 + \left(\frac{2}{7}x^2 - \frac{6}{7}x^2\right) + \frac{1}{7}x + \left(\frac{3}{7} - \frac{2}{7}\right) \\ = -\frac{4}{7}x^3 - \frac{4}{7}x^2 + \frac{1}{7}x + \frac{1}{7}$$

This is cubic polynomial.

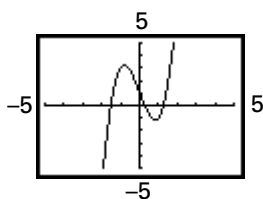
$$54. \quad m(x) = x^4 - 10x^2 + 9$$



The graph of this quartic function is a W-shape with 3 turns.

### Lesson 7.2

$$11. \quad P(x) = 2x^3 - 4x + 1$$

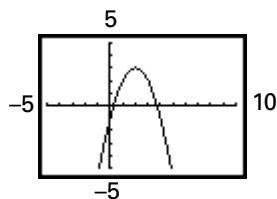


Maximum of  $\approx 3.2$

Minimum of  $\approx -1.2$

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23.  $P(x) = -x^2 + 4x - 1$



Local maximum of  $\approx 3.0$

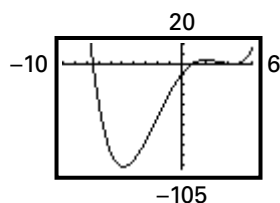
No local minimum

The function increases for  $x < 2.0$   
and decreases for  $2.0 < x$ .

31.  $P(x) = 6x + 1 - x^2$  or  $P(x) = -x^2 + 6x + 1$

The degree of the function is even and the leading coefficient is negative. Therefore, the end behavior of the function is that the graph falls on the left and the right.

40. Make a scatter plot of the data points and find the quartic regression model.



The data in the table can be modeled by the quartic function  
 $f(x) \approx 0.08x^4 - 0.17x^3 - 3.58x^2 + 14.67x - 11$ .

### Lesson 7.3

15.  $2x^3(4x^3 - 2x^2 + x + 3)$   
 $= 8x^6 - 4x^5 + 2x^4 + 6x^3$

27.  $(2x - 4)(x + 1)^2 = (2x - 4)(x^2 + 2x + 1)$   
 $= (2x - 4)(x^2) + (2x - 4)(2x) + (2x - 4)(1)$   
 $= 2x^3 - 4x^2 + 4x^2 - 8x + 2x - 4$   
 $= 2x^3 - 6x - 4$

35.  $x^3 + 6x^2 + 8x = x(x^2 + 6x + 8)$   
 $= x(x + 2)(x + 4)$



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45.  $x^5 - 9x^3 + 8x = 0$   
 $x(x^4 - 9x^2 + 8) = 0$   
 $x = 0$  or  $x^4 - 9x^2 + 8 = 0$

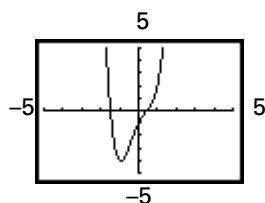
Substitute  $u$  for  $x^2$  and solve for  $u$ .

$$\begin{aligned} x^4 - 9x^2 + 8 &= 0 \\ (x^2)^2 - 9x^2 + 8 &= 0 \\ u^2 - 9u + 8 &= 0 \\ (u - 8)(u - 1) &= 0 \end{aligned}$$

Replace  $u$  with  $x^2$  and solve for  $x$ .

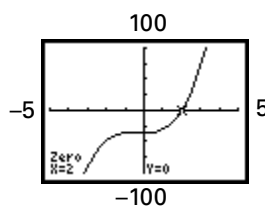
$$\begin{aligned} (x^2 - 8)(x^2 - 1) &= 0 \\ x^2 = 8 \quad \text{or} \quad x^2 = 1 \\ x = \pm\sqrt{8} \quad \text{or} \quad x = \pm 1 \\ x = 0 \quad \text{or} \quad x = \pm 2\sqrt{2} \quad \text{or} \quad x = \pm 1 \end{aligned}$$

57.  $g(x) = 2x^4 - 2x^2 + 3x - 1$



There are zeros of  $g(x)$  between  $-2$  and  $-1$  (about  $-1.49$ ) and between  $0$  and  $1$  (about  $0.44$ ).

63.  $3x^3 + 3x^2 = 36$   
 $3x^3 + 3x^2 - 36 = 0$

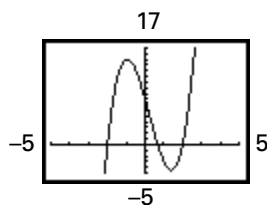


The width is  $2$ , so the length is  $3(2) = 6$  and the height is  $1 + 2 = 3$ . The dimensions are  $2$  feet by  $6$  feet by  $3$  feet.

## Lesson 7.5

13.  $3x^3 - 2x^2 - 12x + 8 = 0$

Possible rational zeros are all the quotients  $\frac{p}{q}$  that can be formed from  $p = \pm 1, \pm 2, \pm 4, \pm 8$  and  $q = \pm 1, \pm 3$ . Use a graph to narrow the possibilities.

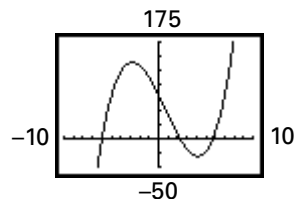


From the graph, there are zeros at  $-2$  and  $2$ . There is also a zero between  $0$  and  $1$ . Use synthetic division to test possible zeros that are between these values.

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & -12 & 8 \\ & & 2 & 0 & -8 \\ \hline & 3 & 0 & -12 & 0 \end{array}$$

The rational roots are  $-2$ ,  $2$ , and  $\frac{2}{3}$ .

26.  $t(x) = x^3 - 2x^2 + 78 - 35x$



From the graph, there is a zero at  $-6$ .

Then  $x + 6$  is a factor of  $t(x)$ .

$$x^3 - 2x^2 + 78 - 35x = 0$$

$$(x + 6)(x^2 - 8x + 13) = 0$$

$$x + 6 = 0 \text{ or } x^2 - 8x + 13 = 0$$

$$x = -6 \qquad x = \frac{8 \pm \sqrt{64 - 52}}{2}$$

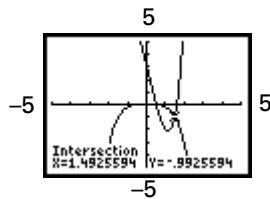
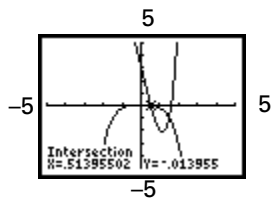
$$x = \frac{8 \pm 2\sqrt{3}}{2}$$

$$x = 4 \pm \sqrt{3}$$

The zeros of  $t(x)$  are  $-6$ ,  $4 + \sqrt{3}$ , and  $4 - \sqrt{3}$ .

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36. Graph  $P(x) = x^4 - 6x + 3$  and  $Q(x) = -0.2x^4$  in the same window.



The graphs intersect twice, at  $x \approx 0.51$  and  $x \approx 1.49$ .

45.  $P(x) = a(x - 1)^2(x - 2)^2$   
 $1 = a(0 - 1)^2(0 - 2)^2$   
 $1 = a(-1)^2(-2)^2$   
 $1 = 4a$   
 $a = \frac{1}{4}$   
 $P(x) = \frac{1}{4}(x - 1)^2(x - 2)^2$   
 $= \frac{1}{4}x^4 - \frac{3}{2}x^3 + \frac{13}{4}x^2 - 3x + 1$