

**Chapter 6 Parent Guide**  
**Exponential and Logarithmic Functions**

**In an exponential function, the variable is** at least a part of the exponent. Logarithms are used to determine the value of the exponent.

Biologists use exponential functions and logarithms to study and analyze bacterial growth and decay. Physicists use them in the study of sound and chemists use them to study chemical reactions. Many other applications, including financial growth, are also possible.

Though your child has used exponents extensively in other courses, including algebra and geometry, the terminology and exponential notation in Chapter 6 will be new. With study, your child will recognize that the rules for exponents and logarithms are the same.

Lesson 6.1 introduces the chapter with a real-life application of exponential growth and decay. Lesson 6.2 classifies exponential functions and calculates the growth of investments under various conditions. Lesson 6.3 introduces logarithmic functions and how they relate to an exponential function. Lesson 6.4 discusses the properties of logarithmic functions. Lesson 6.5 uses logarithms to solve exponential and logarithmic equations. Lesson 6.6 discusses natural logarithms and how their applications differ from logarithms of base 10. Lesson 6.7 solves logarithmic and exponential equations.

The following activity will help you and your child become comfortable with the topics in this chapter.

**PROBLEM FOR DISCUSSION (See textbook page 362)**

You can use exponential functions to calculate the value of investments that earn compound interest and to compare different investments by calculating effective yields.

Consider the function  $y = x^2$  and  $y = 2^x$ . Both functions have a base and an exponent. However  $y = x^2$  is a quadratic function, and  $y = 2^x$  is an exponential function. In an exponential function, the base is fixed and the exponent is variable. What types of patterns can you expect from an exponential function?

1. Discuss how the equations  $y = x^2$  and  $y = 2^x$  compare. How are they alike? How are they different?

Both of these equations involve exponents. The difference is that in  $y = x^2$ , each value for  $x$  is squared. In  $y = 2^x$ , 2 is raised to the power of  $x$ . The difference in these two equations can be more easily seen in a table of their values.

$y = x^2$

$x$	0	1	2	3	4	5	6	7	8	9
$y$	0	1	4	9	16	25	36	49	64	81

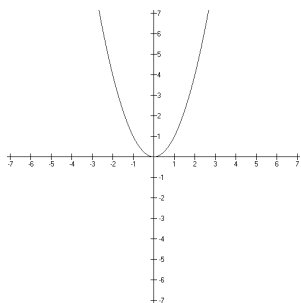
$y = 2^x$

$x$	0	1	2	3	4	5	6	7	8	9
$y$	1	2	4	8	16	32	64	128	256	512

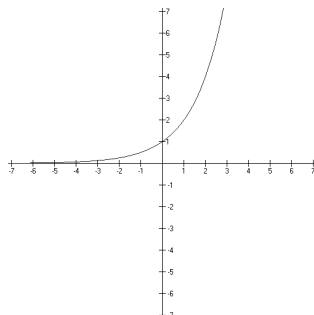
2. Discuss how the graphs of  $y = x^2$  and  $y = 2^x$  differ from one another. How are they alike?

Consider the graphs of each:

$$y = x^2$$



$$y = 2^x$$



The graphs of both equations increase. The graph of  $y = 2^x$  increases at a faster rate. The other difference is that the graph of  $y = 2^x$  is not defined for values of  $x$  less than zero.

3. Look at the graphs of  $f(x)$  and  $g(x)$  on page 363. Where do they intersect? Discuss why all exponential functions of the form  $h(x) = b^x$  will pass through  $(0, 1)$ .

The graphs of  $f(x)$  and  $g(x)$  intersect at  $(0, 1)$ .

All exponential functions of the form  $h(x) = b^x$  will pass through  $(0, 1)$  because any number raised to the zero power is 1.

4. Predict the value for  $y$  in  $(0, y)$  if  $k(x) = b^{ax+c}$ .

Substitute 0 in for  $x$  and try to figure out what  $y$  is.

$$k(x) = b^{a(0)+c}$$

$$k(x) = b^{0+c}$$

$$k(x) = b^c$$

So,  $(0, y)$  is  $(0, b^c)$ .

5. What have you learned about exponential functions from this activity?

Exponential functions increase rapidly, intersect at  $(0, 1)$ , and are not defined for  $x$  less than zero.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

### Lesson 6.1

17.  $1 - 6\% = 1 - 0.06 = 0.94$

25.  $3^y = 3^{\frac{3}{5}} \approx 1.9$

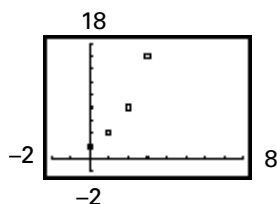
36.  $55(2)^x$

a.  $55(2)^3 = 440$  bacteria

b.  $55(2)^5 = 1760$  bacteria

43. Plot the points.

$x$	0	1	2	3
$y$	2	4	8	16



The graph reveals that there is an exponential relationship between  $x$  and  $y$ .

52. a. If 9 milliliters of the buffer is added to 1 milliliter of the already diluted mixture, the concentration of the resulting mixture is  $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$  of the original concentration of HCl.

b.  $\left(\frac{1}{10}\right)^n$

c.  $\left(\frac{1}{10}\right)^n = \left(\frac{1}{10}\right)^5 = \frac{1}{100,000}$

After 5 dilutions, the concentration is  $\frac{1}{100,000}$  of the original concentration of HCl.

### Lesson 6.2

16.  $y(x) = 12(2.5)^x$  represents exponential growth because  $b > 1$  ( $b = 2.5$ ).

28.  $y = \left(\frac{1}{2}\right)^x$  is graph a. The  $y$ -intercept is 1 and it is an exponential decay function.

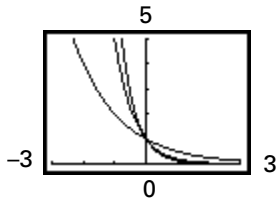
31.  $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

$A(10) = 750\left(1 + \frac{0.10}{4}\right)^{4 \cdot 10} \approx 2013.80$

The final amount is \$2013.80.

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36.  $a(x) = \left(\frac{1}{2}\right)^x$ ,  $b(x) = \left(\frac{1}{5}\right)^x$ ,  $c(x) = \left(\frac{1}{8}\right)^x$



- a. The function  $c(x) = \left(\frac{1}{8}\right)^x$  exhibits the fastest decay, whereas  $a(x) = \left(\frac{1}{2}\right)^x$  exhibits the slowest decay.
- b. The  $y$ -intercepts are all equal at  $y = 1$ .
- c. The domains and ranges are the same for each function.  
domain: all real numbers  
range: all positive real numbers

51.  $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$   
 $A(5) = 2000\left(1 + \frac{0.0551}{1}\right)^{1 \cdot 5} \approx 2615.16$   
The final amount is \$2615.16.

### Lesson 6.3

18.  $27^{\frac{1}{3}} = 3$   
 $\log_{27} 3 = \frac{1}{3}$

27.  $\log_{10} 0.1 = -1$   
 $10^{-1} = 0.1$

35.  $\log_{144} 12 = \frac{1}{2}$   
 $144^{\frac{1}{2}} = 12$

43.  $\log_{10} 0.00013 \approx -4$

69.  $2 = \log_7 v$   
 $v = 7^2 = 49$

92. reflection across the  $x$ -axis and vertical stretch  
by a factor of 5

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### Lesson 6.4

$$\begin{aligned} 18. \log_2 35 &= \log_2 5 + \log_2 7 \\ &\approx 2.3219 + 2.8074 \\ &\approx 5.1293 \end{aligned}$$

$$30. \log_4 8 + \log_4 2 = \log_4 16 = 2$$

$$45. \log_4 4^5 = 5$$

$$58. \log_{10}(5x - 3) - \log_{10}(x^2 + 1) = 0$$

$$\begin{aligned} \log_{10} \frac{5x - 3}{x^2 + 1} &= 0 \\ \frac{5x - 3}{x^2 + 1} &= 10^0 \\ \frac{5x - 3}{x^2 + 1} &= 1 \\ 5x - 3 &= x^2 + 1 \\ x^2 - 5x + 4 &= 0 \\ (x - 4)(x - 1) &= 0 \end{aligned}$$

$$x = 1 \text{ or } x = 4$$

Check:

$$\text{Let } x = 4$$

$$\log_{10}(20 - 3) - \log_{10}(16 + 1) = 0$$

$$\log_{10} 17 - \log_{10} 17 = 0 \text{ True}$$

$$\text{Let } x = 1$$

$$\log_{10}(5 - 3) - \log_{10}(1 + 1) = 0$$

$$\log_{10} 2 - \log_{10} 2 = 0 \text{ True}$$

$$65. \log x^2 = 2 \log x$$

By the Power Property of Logarithms,

$\log x^2 = 2 \log x$ ; this equation is always true.

$$74. \log_{10} S = 0.425 \log_{10} W + 0.725 \log_{10} H + \log_{10} 71.84$$

$$\log_{10} S = \log_{10}(W^{0.425} H^{0.725} \cdot 71.84)$$

$$S = 71.84 W^{0.425} H^{0.725}$$

### Lesson 6.5

$$14. 3.5^x = 28$$

$$\log 3.5^x = \log 28$$

$$x \log 3.5 = \log 28$$

$$x = \frac{\log 28}{\log 3.5} \approx 2.66$$

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33.  $\log_5 2 = \frac{\log 2}{\log 5} \approx 0.43$

50.  $R = 10 \log \frac{I}{I_0}$   
 $55 = 10 \log \frac{I}{I_0}$   
 $5.5 = \log \frac{I}{I_0}$   
 $\frac{I}{I_0} = 10^{5.5}$

The intensity of the engine is  $10^{5.5}$  times louder than the threshold of hearing.

56.  $\text{pH} = -\log [\text{H}^+]$   
 $0 = -\log [\text{H}^+]$   
 $0 = \log [\text{H}^+]$   
 $[\text{H}^+] = 10^0 = 1$   
 $7 = \log [\text{H}^+]$   
 $-7 = \log [\text{H}^+]$   
 $[\text{H}^+] = 10^{-7}$

The range is  $10^{-7} < [\text{H}^+] < 10^0 = 1$ .

## Lesson 6.6

18.  $2e^{-0.5} \approx 1.213$

34.  $e^{2.5} \approx 12.182$   
 $\ln 2.5 \approx 0.916$   
 $10^{2.5} \approx 316.228$   
 $\log 2.5 \approx 0.398$   
 $\log 2.5, \ln 2.5, e^{2.5}, 10^{2.5}$

38.  $e^{6x-4} = e^{6x} \cdot e^{-4}$   
By Properties of Exponents,  
 $e^{6x} \cdot e^{-4} = e^{6x+(-4)} = e^{6x-4}$ ;  
this equation is always true.

51.  $\ln 5 \approx 1.61$   
 $e^{\ln 5} \approx e^{1.61}$   
 $5 \approx e^{1.61}$

69.  $h$  to  $f$ : reflection across the  $y$ -axis and horizontal compression by a factor of  $\frac{1}{2}$   
 $h$  to  $g$ : reflection across the  $y$ -axis  
 $h$  to  $i$ : horizontal compression by a factor of  $\frac{1}{2}$

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74.  $A = Pe^{rt}$

$r = 3\%$

$A = 2000e^{0.03 \cdot 20} \approx 3644.24$

$r = 5\%$ :

$A = 2000e^{0.05 \cdot 20} \approx 5436.56$

The investment at 5% grows much faster than the investment at 3%.

### Lesson 6.7

11.  $x = \log_3 \frac{1}{27}$

$3^x = \frac{1}{27}$

$3^x = 3^{-3}$

$x = -3$

19.  $10^{2x} + 75 = 150$

$10^{2x} = 75$

$2x = \log 75$

$x = \frac{\log 75}{2} \approx 0.94$

29.  $M = \frac{2}{3} \log \frac{E}{10^{11.8}}$

$= \frac{2}{3} \log \frac{8 \times 10^{19}}{10^{11.8}} \approx 5.4$

32.  $a(t) = 82 - 12 \log(t + 1)$

a. When  $t = 0$ ,

$a(0) = 82 - 12 \log(0 + 1)$

$= 82 - 0$

$= 82$

The original average score was 82.

b. When  $t = 6$ ,

$a(6) = 82 - 12 \log(6 + 1)$

$\approx 72$

After 6 months, the average score was 72.