

Chapter 5 Parent Guide

Quadratic Functions

After studying real numbers for so long, students have asked the question, Are there unreal numbers?

The answer is yes, but they are called complex numbers.

Quadratic functions are second-degree equations. Chapter 5 reviews solving various types of quadratic equations, building to the use of the quadratic formula. Up until now, all numbers inside the radical symbol of the quadratic formula have been positive. Taking the square root of a negative number results in a complex number.

This chapter also introduces fitting a curve to data and solving quadratic inequalities. Lesson 5.1 introduces quadratic functions and determines their minimum or maximum value. Lesson 5.2 solves certain quadratic equations by taking the square root of both sides.

Lesson 5.3 reviews factoring quadratic expressions. Lesson 5.4 uses completing the square to solve a quadratic expression. Lesson 5.5 uses the quadratic formula to analyze the graph of a function and find the real roots. Lesson 5.6 uses the discriminant to classify a quadratic equation. It includes how to perform operations on complex numbers. Lesson 5.7 discusses using a graphics calculator to fit a curve to a scatter plot. Lesson 5.8 uses a graphics calculator to solve quadratic inequalities.

This chapter prepares students for analyzing exponential and logarithmic functions in Chapter 6.

Go through the following activity with your child to confirm an understanding of the difference between quadratic and linear functions.

PROBLEM FOR DISCUSSION (See textbook page 274)

Recall from Lesson 2.4 that the total stopping distance of a car on certain types of road surfaces is modeled by the function

$$d(x) = \frac{11}{10}x + \frac{1}{19}x^2$$

where x is the speed of the car in miles per hour at the moment the hazard is observed and $d(x)$ is the distance in feet required to bring the car to a complete stop.

1. Discuss the information in the paragraph at the bottom of page 274.

The information at the bottom of page 274 discusses whether or not $d(x)$ is a linear function or not. If $d(x)$ is linear, then the stopping distance for any speed would be proportional. This is not the case. As the speed increases at a constant rate, the stopping distance increases much faster. Therefore, it is not a linear relationship.

2. If the function were linear and a car traveling 20 miles an hour requires 43 feet to come to a complete stop, how many feet would be needed to come to a complete stop at 60 miles per hour?

If $d(x)$ was a linear relationship, then the values would be proportional. So, you can set up a proportion and solve for the number of feet needed.

$$\frac{20}{43} = \frac{60}{y}$$

$$20y = 2580 \quad \text{Cross-multiply}$$

$$y = 129 \quad \text{Divide both sides by 20 to solve for } y$$

So, a car driving 60 miles per hour would need 129 feet to stop if $d(x)$ was linear.

3. Discuss how you can recognize a quadratic function from its equation. How does it differ from a cubic equation?

You can easily recognize a quadratic function from its equation. A quadratic equation has the form $y = ax^2 + bx + c$. The largest degree (look at the value of the exponent) of any term in a quadratic is two.

A cubic equation's largest term has a degree of three and usually is in the form

$$y = ax^3 + bx^2 + cx + d.$$

4. Discuss what you have learned about quadratic functions from this activity.

You have learned that quadratic functions do not have the same relationship as linear functions. You have also learned some differences between a quadratic function and a cubic function.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

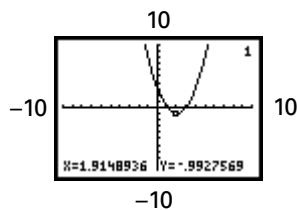
Lesson 5.1

13. $f(x) = (x - 3)(x + 8)$
 $f(x) = x^2 - 3x + 8x - 24$
 $f(x) = x^2 + 5x - 24$
 $a = 1, b = 5, \text{ and } c = -24$

28. $g(s) = 3 - s = -s + 3$
 $g(s)$ is not a quadratic function
 since it cannot be written
 in the standard form
 $g(s) = as^2 + bs + c.$

34. $f(x) = 8x^2 - x$
 Since $a = 8 > 0$, the
 parabola opens up and the
 y -coordinate of the vertex is
 the minimum value of the
 function.

45. $f(x) = (x - 2)^2 - 1$
 $= (x^2 - 4x + 4) - 1$
 $= x^2 - 4x + 3$



The vertex is approximately $x = 1.9, y = -1$.

Lesson 5.2

14. $x^2 = 121$
 $x = \pm\sqrt{121}$
 $x = -\sqrt{121} \text{ or } x = \sqrt{121}$
 $x = -11 \text{ or } x = 11$

33. $r^2 + s^2 = t^2$
 $0.8^2 + 2^2 = t^2$
 $t = \sqrt{0.8^2 + 2^2}$
 $t \approx 2.2$

40. $a^2 + b^2 = c^2$
 $a^2 + 3^2 = 5^2$
 $a^2 = 5^2 - 3^2$
 $a = \sqrt{5^2 - 3^2}$
 $a = 4$

Chapter 5

- 49. a.** The diagonal along one face is the hypotenuse of a right triangle with the other two legs being edges of the cube.

$$a^2 = 3^2 + 3^2$$

$$a^2 = 2 \cdot 3^2$$

$$a = \sqrt{18}$$

$$a = 3\sqrt{2}$$

$$a \approx 4.24 \text{ feet}$$

- b.** The diagonal passing through the interior of the cube is the hypotenuse of a right triangle with the other two legs being sides of length a and 3.

$$\text{Then } b^2 = a^2 + 3^2$$

$$b^2 = a^2 + 9$$

$$b^2 = (3\sqrt{2})^2 + 9$$

$$b^2 = 18 + 9$$

$$b = \sqrt{27}$$

$$b = 3\sqrt{3}$$

$$b \approx 5.20 \text{ feet}$$

Lesson 5.3

32. $10n - n^2 = n \cdot 10 - n \cdot n$
 $= n(10 - n)$

47. $2x + x^2 - 24 = x^2 + 2x - 24$
 $= (x + 6)(x - 4)$

54. $2x^2 + 3x + 1 = (2x + 1)(x + 1)$

63. $6x^2 - 17x = -12$
 $6x^2 - 17x + 12 = 0$
 $(3x - 4)(2x - 3) = 0$
 $3x - 4 = 0$ or $2x - 3 = 0$
 $3x = 4$ or $2x = 3$
 $x = \frac{4}{3}$ or $x = \frac{3}{2}$

72. $9x^2 = -6x - 1$
 $9x^2 + 6x + 1 = 0$
 $(3x + 1)^2 = 0$
 $3x + 1 = 0$
 $3x = -1$
 $x = -\frac{1}{3}$

Chapter 5

- 100.** Let the length of the shorter leg be x . Then the longer leg has length $x + 7$.

$$\begin{aligned}x^2 + (x + 7)^2 &= 13^2 \\x^2 + x^2 + 14x + 49 &= 169 \\2x^2 + 14x - 120 &= 0 \\2(x^2 + 7x - 60) &= 0 \\2(x + 12)(x - 5) &= 0 \\x + 12 = 0 \quad \text{or} \quad x - 5 = 0 \\x = -12 \quad \text{or} \quad x = 5\end{aligned}$$

The solution $x = -12$ does not make sense here so the only solution is $x = 5$. The triangle has legs of length 5 cm and 12 cm.

Lesson 5.4

- 12.** $x^2 - 14x$

$$\begin{aligned}\frac{1}{2}(-14) &= -7 \rightarrow (-7)^2 = 49 \\x^2 - 14x + 49 &= (x - 7)^2\end{aligned}$$

- 19.** $x^2 - 5x - 1 = 4 - 3x$
 $x^2 - 2x = 5$

$$\begin{aligned}x^2 - 2x + \left(\frac{-2}{2}\right)^2 &= 5 + \left(\frac{-2}{2}\right)^2 \\(x - 1)^2 &= 6 \\x - 1 &= \pm\sqrt{6} \\x - 1 = -\sqrt{6} \quad \text{or} \quad x - 1 = \sqrt{6} \\x = 1 - \sqrt{6} \quad \text{or} \quad x = 1 + \sqrt{6}\end{aligned}$$

- 38.** $g(x) = 3x^2$

$$g(x) = 3(x^2)$$

$$g(x) = 3(x - 0)^2 + 0$$

The vertex is at $(0, 0)$ and
the axis of symmetry has
the equation $x = 0$.

- 49.** Let x be the length of the side of the original square. If it is increased by 2 cm, the new length is $x + 2$ cm.

$$\begin{aligned}A &= 30 \\(x + 2)^2 &= 30 \\x + 2 &= \pm\sqrt{30} \\x &= -2 \pm \sqrt{30} \\x + 2 = -\sqrt{30} \quad \text{or} \quad x + 2 = \sqrt{30} \\x = -2 - \sqrt{30} \quad \text{or} \quad x = -2 + \sqrt{30} \\x \approx -7.5 \quad \text{or} \quad x \approx 3.5\end{aligned}$$

A negative length is not possible, so the original square had sides of length $-2 + \sqrt{30} \approx 3.5$ cm.

Chapter 5

Lesson 5.5

13. $(x - 4)(x + 5) = 7$

$$x^2 + x - 20 = 7$$

$$x^2 + x - 27 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-27)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{109}}{2}$$

$$x = \frac{-1 - \sqrt{109}}{2} \quad \text{or} \quad x = \frac{-1 + \sqrt{109}}{2}$$

25. $5x^2 - 2x - 3 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-3)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{64}}{10}$$

$$x = \frac{2 \pm 8}{10}$$

$$x = \frac{2 - 8}{10} \quad \text{or} \quad x = \frac{2 + 8}{10}$$

$$x = -\frac{3}{5} \quad \text{or} \quad x = 1$$

32. $x = -\frac{6}{2(3)}$ and $y = 3x^2 + 6x - 18$
 $x = -1$ and $y = 3(-1)^2 + 6(-1) - 18$
 $y = -21$

Thus, the equation for the axis of symmetry is $x = -1$, and the coordinates of the vertex are $(-1, -21)$.

37. $x = -\frac{3}{2(-2)}$ and $y = -2x^2 + 3x - 1$
 $x = \frac{3}{4}$ and $y = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) - 1$
 $y = \frac{1}{8}$

Thus, the equation for the axis of symmetry is $x = \frac{3}{4}$, and the coordinates of the vertex are $\left(\frac{3}{4}, \frac{1}{8}\right)$.

52. The depth of the rain gutter is the same as the amount of aluminum that is bent up on each side, x . The width of the rain gutter will be $12 - 2x$.

$$\text{area} = \text{depth} \times \text{width}$$

$$A(x) = x \cdot (12 - 2x)$$

$$18 = 12x - 2x^2$$

Chapter 5

Lesson 5.6

16. $6 = 6 + 0i$: 6 is the real part, 0 is the imaginary part.

$$\begin{aligned} 36. \quad & 16 - 8x = -x^2 \\ & x^2 - 8x + 16 = 0 \\ & b^2 - 4ac = (-8)^2 - 4(1)(16) \\ & \quad = 64 - 64 \\ & \quad = 0 \end{aligned}$$

Since $b^2 - 4ac = 0$ the equation has 1 real solution.

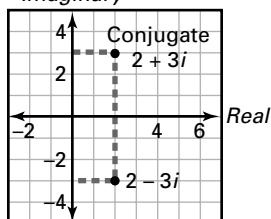
$$\begin{aligned} & x^2 - 8x + 16 = 0 \\ & (x - 4)^2 = 0 \\ & x = 4 \end{aligned}$$

$$\begin{aligned} 49. \quad (3 + i) + (6 - 2i) &= (3 + 6) + (i - 2i) \\ &= 9 - i \end{aligned}$$

$$\begin{aligned} 56. \quad (2 - 4i)^2 &= (2 - 4i)(2 - 4i) \\ &= 2 \cdot (2 - 4i) - 4i \cdot (2 - 4i) \\ &= 4 - 8i - 8i + 16i^2 \\ &= -12 - 16i \end{aligned}$$

$$\begin{aligned} 70. \quad (1 + i)^2 + \frac{2 + 2i}{2 + i} &= (1 + i) \cdot (1 + i) + \frac{2 + 2i}{2 + i} \cdot \frac{2 - i}{2 - i} \\ &= 1 + i + i + i^2 + \frac{(2 + 2i)(2 - i)}{(2 + i)(2 - i)} \\ &= \frac{2i + 4 - 2i + 4i - 2i^2}{4 - 2i + 2i - i^2} \\ &= 2i + \frac{6 + 2i}{5} \\ &= \frac{6}{5} + \frac{12}{5}i \end{aligned}$$

79. *Imaginary*



Chapter 5

Lesson 5.7

14. $y = ax^2 + bx + c$

$$\begin{aligned} (1, 7): & \quad 7 = a + b + c \\ (-2, 4) & \quad 4 = 4a - 2b + c \\ (3, 19): & \quad 19 = 9a + 3b + c \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & -2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 19 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$y = x^2 + 2x + 4$$

22. a.

Cube	Unit cubes with exactly 3 green faces	Unit cubes with exactly 2 green faces	Unit cubes with exactly 1 green face
2	8	0	0
3	8	12	6
4	8	24	24
5	8	36	54
6	8	48	96

b.

n	8	$12(n - 2)$	$6(n - 2)^2$
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c.

7	8	60	150
8	8	72	216
20	8	216	1944

23. $h(t) = at^2 + bt + c$

$$\begin{aligned} (1, 19): & \quad 19 = a + b + c \\ (2, 21): & \quad 21 = 4a + 2b + c \\ (3, 11): & \quad 11 = 9a + 3b + c \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 19 \\ 21 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -6 \\ 20 \\ 5 \end{bmatrix}$$

$$h(t) = -6t^2 + 20t + 5$$

Chapter 5

29. $h(t) = 16$
 $-6t^2 + 20t + 5 = 16$
 $-6t^2 + 20t - 11 = 0$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-6)(-11)}}{2(-6)}$$

$$= \frac{-20 \pm \sqrt{136}}{-12}$$

$$= \frac{5}{3} \pm \frac{\sqrt{34}}{6}$$

$$t \approx 0.69 \text{ or } t \approx 2.64$$

The object is 16 feet above the surface after 0.69 seconds and again after 2.64 seconds.

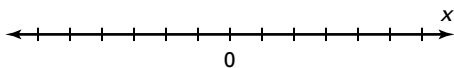
Lesson 5.8

20. $-x^2 + \frac{4}{3}x - \frac{5}{9} > 0$
 $-x^2 + \frac{4}{3}x - \frac{5}{9} = 0$

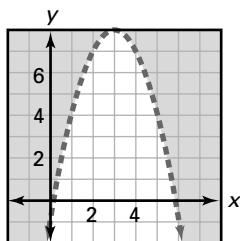
No real zeros.

Test $x = 0, f(0) = -\frac{5}{9}$: $-\frac{5}{9} \not\geq 0$ False

Therefore, $-x^2 + \frac{4}{3}x - \frac{5}{9} > 0$ has no solution.



37.



$A(5, 1): 1 \not\geq -(5 - 3)^2 + 8 = 4$ False

$B(5, 4): 4 \not\geq -(5 - 3)^2 + 8 = 4$ False

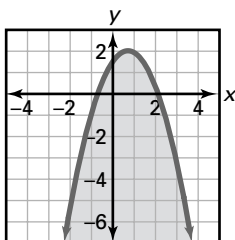
$C(5, 6): 6 \geq -(5 - 3)^2 + 8 = 4$ True

C is in the solution region.

53. $y \leq -(x - \frac{5}{7})^2 + 2$

Test $(0, 0)$:

$0 \leq -(0 - \frac{5}{7})^2 + 2 = \frac{73}{49}$ True



Chapter 5

62. a. $p(x) > 0$
 $-0.1x^2 + 8x - 50 > 0$
 $-0.1x^2 + 8x - 50 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(-0.1)(-50)}}{-0.2}$$

$$x = \frac{-8 \pm \sqrt{44}}{-0.2}$$

 $x \approx 6.83$ or $x \approx 73.17$

To make a profit, at least 7 bumper stickers must be sold.

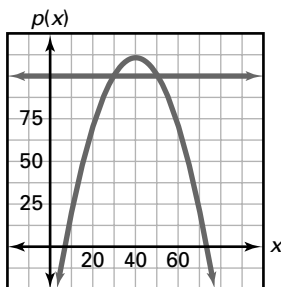
b. $p(x) > 100$
 $-0.1x^2 + 8x - 50 > 100$
 $-0.1x^2 + 8x - 150 > 0$
 $-0.1x^2 + 8x - 150 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(-0.1)(-150)}}{-0.2}$$

$$x = \frac{-8 \pm 2}{-0.2}$$

 $x = 30$ or $x = 50$

Yes, the profit is greater than \$100 for $30 < x < 50$.



Examine the graph and note that points lie above the horizontal line $y = 100$ for $30 < x < 50$.