Chapter 4 Parent Guide
Matrices

A matrix is a modified table used to analyze and compute the data stored in it. It allows several groups of data to be considered simultaneously. Matrices can be any rectangular shape or size. People who use matrices include computer programmers and business analysts.

Because a matrix is a kind of mathematical notation, it comes with its own set of rules. Students will need to learn how to compute with matrices and use them to solve systems of equations.

Lesson 4.1 shows how to use matrices to represent data. Lesson 4.2 involves multiplying two matrices and identifying matrices that can and cannot be multiplied. Lesson 4.3 illustrates finding and using the inverse of a matrix.

Lesson 4.4 involves using matrices to solve systems of linear equations. Lesson 4.5 is about representing a system of equations as an augmented matrix.

Students who are visually perceptive will have an edge over those who are not in this chapter. Computing with matrices requires organized spatial thinking.

Once mastered, your child will find matrices an excellent tool for solving complex systems of equations sometimes required in subsequent mathematics and physics courses.

You can help your child to develop his or her spatial thinking skills by doing the following activity together. It discusses how to multiply two matrices with compatible dimensions.

PROBLEM FOR DISCUSSION (See textbook page 225)

A football team scores 5 touchdowns, 4 extra points, and 2 field goals. A touchdown is worth 6 points, an extra point is 1 point, and a field goal is 3 points. The final score is evaluated as follows:

(5 touchdowns)(6 pts) +(4 extra points)(1 pt)+(2 field goals)(3 pts)
= 30 points + 4 points + 6 points
= 40 points total

How can this data be represented in a matrix?

1. Discuss how the total points are found in a football game. What operations are used to find the total points.

   Total points are found by adding up all the different types of points that can be scored in a game.

   To figure out the total points, multiply the number of touchdowns by the point value of a touchdown, the number of extra points by the value of an extra point, and the number of field goals by the value of a field goal.

2. Discuss how the data is represented in the two matrices shown on page 225.

The number of touchdowns, extra points, and field goals are in the horizontal matrix $[5 \quad 4 \quad 2]$.

The value of touchdowns, extra points, and field goals are in the vertical matrix $\begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix}$.

3. How are the two matrices the same? How are they different?

They are the same in that they both have three values in each matrix.

They are different because the dimensions of each matrix are different. One of the matrices is a 1×3 matrix and the other is a 3×1 matrix.

4. How does computing the total score, above, compare to computing the score by using matrices?

Computing the score either way results in the same answer. The process for multiplying matrices is actually the same process as shown above. In matrix multiplication you would multiply (5)(6) and then add that to the product of (4)(1) and then finally add that to the product of (2)(3).

5. What are the advantages to computing with matrices?

Sometimes when computing with a large group of data, numbers may get "lost" or overlooked. Computing with matrices is a straightforward step by step process of organizing and computing data.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

Chapter 4 - Solutions to Selected Exercises from the Student Textbook

Lesson 4.1

19. $-4C = \begin{bmatrix} -4(7) \\ -4(2) \\ -4(6) \end{bmatrix} = \begin{bmatrix} -28 \\ -8 \\ -24 \end{bmatrix}$

27. $\begin{bmatrix} \frac{2}{3}x & 12 \\ -4 & \frac{1}{2}y + 5 \end{bmatrix} = \begin{bmatrix} 6 & x + 3 \\ -4 & y + 1 \end{bmatrix}$

$\frac{2}{3}x = 6$

$x = 9$

$12 = x + 3$

$x = 9$

$\frac{1}{2}y + 5 = y + 1$

$y + 10 = 2y + 2$

$8 = y$

$x = 9$ and $y = 8$

40. $-\left(\frac{1}{2}B - A\right) = -\frac{1}{2}B + A$

$$= \begin{bmatrix} -\frac{1}{2}(6) & -\frac{1}{2}(0) & -\frac{1}{2}(11) & -\frac{1}{2}(-3) \\ -\frac{1}{2}(-5) & -\frac{1}{2}(2) & -\frac{1}{2}(-8) & -\frac{1}{2}(9) \end{bmatrix} + \begin{bmatrix} 7 & 3 & -1 & 5 \\ -2 & 8 & 0 & -4 \end{bmatrix}$$

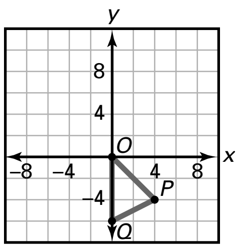
$$= \begin{bmatrix} -3 + 7 & 0 + 3 & -\frac{11}{2} + (-1) & \frac{3}{2} + 5 \\ \frac{5}{2} + (-2) & -1 + 8 & 4 + 0 & -\frac{9}{2} + (-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & -\frac{13}{2} & \frac{13}{2} \\ \frac{1}{2} & 7 & 4 & -\frac{17}{2} \end{bmatrix}$$

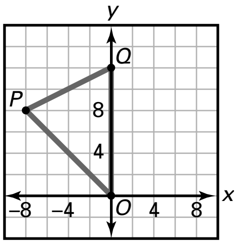
48. a. $O = (0, 0), P = (2, -2), Q = (0, -3)$

$B = \begin{bmatrix} 0 & 2 & 0 \\ 0 & -2 & -3 \end{bmatrix}$

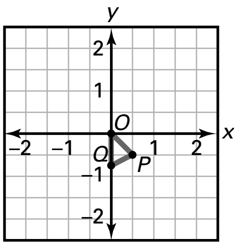
b. $2B = \begin{bmatrix} 2(0) & 2(2) & 2(0) \\ 2(0) & 2(-2) & 2(-3) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 0 & -4 & -6 \end{bmatrix}$



c. $-4B = \begin{bmatrix} -4(0) & -4(2) & -4(0) \\ -4(0) & -4(-2) & -4(-3) \end{bmatrix} = \begin{bmatrix} 0 & -8 & 0 \\ 0 & 8 & 12 \end{bmatrix}$



d. $\frac{1}{4}B = \begin{bmatrix} \frac{1}{4}(0) & \frac{1}{4}(2) & \frac{1}{4}(0) \\ \frac{1}{4}(0) & \frac{1}{4}(-2) & \frac{1}{4}(-3) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{4} \end{bmatrix}$



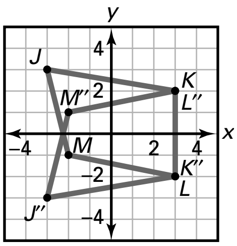
Lesson 4.2

10. $\begin{bmatrix} 1 & 5 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 1(3) + 5(-4) & 1(-2) + 5(6) \\ (-3)(3) + 0(-4) & (-3)(-2) + 0(6) \end{bmatrix} = \begin{bmatrix} -17 & 28 \\ -9 & 6 \end{bmatrix}$

17. *BA* does not exist.

23. a. $BQ = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -3 & 3 & 3 & -2 \\ 3 & 2 & -2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 3 & 3 & -2 \\ -3 & -2 & 2 & 1 \end{bmatrix}$

b.



J'': $(-3, -3)$, K'': $(3, -2)$, L'': $(3, 2)$, M'': $(-2, 1)$

c. Answers may vary. Sample answer: The geometric figure is reflected across the *x*-axis.

Lesson 4.3

14. $\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1(3) - 2(1) = 1$

Yes, the matrix has an inverse.

26. $ad - bc = (-4)(-4) - 8(2) = 0$; no inverse

42. $\begin{bmatrix} -7 & -5 & 2.5 \\ -6 & -4 & 2.5 \\ 3 & 2 & -1 \end{bmatrix}$

Lesson 4.4

11. $\begin{cases} 3x - 5y = 1 \\ 2x + \quad y = -2 \end{cases}$

$$\begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

21. $\begin{cases} 3x + \ 6y - \ 6z = 9 \\ 2x - \ 5y + \ 4z = 6 \\ -x + 16y + 14z = -3 \end{cases}$

$$\begin{bmatrix} 3 & 6 & -6 \\ 2 & -5 & 4 \\ -1 & 16 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 6 & -6 \\ 2 & -5 & 4 \\ 1 & 16 & 14 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{67}{378} & \frac{5}{21} & \frac{1}{126} \\ \frac{8}{189} & -\frac{1}{21} & \frac{2}{63} \\ -\frac{1}{28} & \frac{1}{14} & \frac{1}{28} \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

(3, 0, 0)

32. Let x represent the smallest angle in degrees,
y represent the remaining angle in degrees, and
z represent the largest angle in degrees.

Then $\begin{cases} x + y + z = 180 \\ z = 3x \\ y = \frac{1}{2}(x + z) \end{cases}$

$$\begin{aligned} x + \quad y + z &= 180 \\ 3x \qquad - z &= 0 \\ x - 2y + z &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 180 \\ 0 \\ 0 \end{bmatrix}; \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 60 \\ 90 \end{bmatrix}$$

The angles are 30°, 60°, and 90°.

Lesson 4.5

9. $$\begin{bmatrix} -1 & 2 & -5 & \vdots & 23 \\ 2 & 0 & 7 & \vdots & 19 \\ 5 & -2 & 1 & \vdots & -10 \end{bmatrix}$$

18. $$\begin{bmatrix} 2 & 1 & 2 & \vdots & 19 \\ 3 & 3 & 3 & \vdots & 33 \\ 2 & 2 & 4 & \vdots & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & 1 & \vdots & \frac{19}{2} \\ 3 & 3 & 3 & \vdots & 33 \\ 2 & 2 & 4 & \vdots & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & 1 & \vdots & \frac{19}{2} \\ 0 & \frac{3}{2} & 0 & \vdots & \frac{9}{2} \\ 2 & 2 & 4 & \vdots & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & 1 & \vdots & \frac{19}{2} \\ 0 & \frac{3}{2} & 0 & \vdots & \frac{9}{2} \\ 0 & 1 & 2 & \vdots & 11 \end{bmatrix}$$

$\frac{1}{2}(R_1) \rightarrow R_1$ $\quad -3(R_1) + R_2 \rightarrow R_2$ $\quad -2(R_1) + R_3 \rightarrow R_3$

$$\rightarrow \begin{bmatrix} 1 & \frac{1}{2} & 1 & \vdots & \frac{19}{2} \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 1 & 2 & \vdots & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & \vdots & 8 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 1 & 2 & \vdots & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & \vdots & 8 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 2 & \vdots & 8 \end{bmatrix}$$

$\frac{2}{3}(R_2) \rightarrow R_2$ $\quad -\frac{1}{2}(R_2) + R_1 \rightarrow R_1$ $\quad -1(R_2) + R_3 \rightarrow R_3$

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 & \vdots & 8 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 4 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix}$$

$\frac{1}{2}(R_3) \rightarrow R_3$ $\quad -1(R_3) + R_1 \rightarrow R_1$

24. $$\begin{bmatrix} 2 & -3 & 1 & \vdots & 2 \\ 1 & -1 & 2 & \vdots & 2 \\ 1 & 2 & -3 & \vdots & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & \vdots & 2 \\ 2 & -3 & 1 & \vdots & 2 \\ 1 & 2 & -3 & \vdots & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & \vdots & 2 \\ 0 & -1 & -3 & \vdots & -2 \\ 1 & 2 & -3 & \vdots & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & \vdots & 2 \\ 0 & -1 & -3 & \vdots & -2 \\ 0 & 3 & -5 & \vdots & 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$ $\quad -2(R_1) + R_2 \rightarrow R_2$ $\quad -1(R_1) + R_3 \rightarrow R_3$

$$\rightarrow \begin{bmatrix} 1 & -1 & 2 & \vdots & 2 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 3 & -5 & \vdots & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 5 & \vdots & 4 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 3 & -5 & \vdots & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 5 & \vdots & 4 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & -14 & \vdots & -4 \end{bmatrix}$$

$-1(R_2) \rightarrow R_2$ $\quad R_2 + R_1 \rightarrow R_1$ $\quad -3(R_2) + R_3 \rightarrow R_3$

$$\rightarrow \begin{bmatrix} 1 & 0 & 5 & \vdots & 4 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & \frac{2}{7} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{18}{7} \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & \frac{2}{7} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{18}{7} \\ 0 & 1 & 0 & \vdots & \frac{8}{7} \\ 0 & 0 & 1 & \vdots & \frac{2}{7} \end{bmatrix}$$

$-\frac{1}{14}(R_3) \rightarrow R_3$ $\quad -5(R_3) + R_1 \rightarrow R_1$ $\quad -3(R_3) + R_2 \rightarrow R_2$

$\left(\frac{18}{7}, \frac{8}{7}, \frac{2}{7}\right)$

39. $3p_1 + 18p_2 - 12p_3 = 3$
$p_1 - 2p_2 - 2p_3 = 0$
$p_1 + p_2 + p_3 = 1$

$$\begin{bmatrix} 3 & 18 & -12 & \vdots & 3 \\ 1 & -2 & -2 & \vdots & 0 \\ 1 & 1 & 1 & \vdots & 1 \end{bmatrix} \rightarrow \underset{\frac{1}{3}(R_1) \rightarrow R_1}{\begin{bmatrix} 1 & 6 & -4 & \vdots & 1 \\ 1 & -2 & -2 & \vdots & 0 \\ 1 & 1 & 1 & \vdots & 1 \end{bmatrix}} \rightarrow \underset{-1(R_1) + R_2 \rightarrow R_2}{\begin{bmatrix} 1 & 6 & -4 & \vdots & 1 \\ 0 & -8 & 2 & \vdots & -1 \\ 1 & 1 & 1 & \vdots & 1 \end{bmatrix}} \rightarrow \underset{-1(R_1) + R_3 \rightarrow R_3}{\begin{bmatrix} 1 & 6 & -4 & \vdots & 1 \\ 0 & -8 & 2 & \vdots & -1 \\ 0 & -5 & 5 & \vdots & 0 \end{bmatrix}}$$

$$\rightarrow \underset{-\frac{1}{8}(R_2) \rightarrow R_2}{\begin{bmatrix} 1 & 6 & -4 & \vdots & 1 \\ 0 & 1 & -\frac{1}{4} & \vdots & \frac{1}{8} \\ 0 & -5 & 5 & \vdots & 0 \end{bmatrix}} \rightarrow \underset{-6(R_2) + R_1 \rightarrow R_1}{\begin{bmatrix} 1 & 0 & -\frac{5}{2} & \vdots & \frac{1}{4} \\ 0 & 1 & -\frac{1}{4} & \vdots & \frac{1}{8} \\ 0 & -5 & 5 & \vdots & 0 \end{bmatrix}} \rightarrow \underset{5(R_2) + R_3 \rightarrow R_3}{\begin{bmatrix} 1 & 0 & -\frac{5}{2} & \vdots & \frac{1}{4} \\ 0 & 1 & -\frac{1}{4} & \vdots & \frac{1}{8} \\ 0 & 0 & \frac{15}{4} & \vdots & \frac{5}{8} \end{bmatrix}}$$

$$\rightarrow \underset{\frac{4}{15}(R_3) \rightarrow R_3}{\begin{bmatrix} 1 & 0 & -\frac{5}{2} & \vdots & \frac{1}{4} \\ 0 & 1 & -\frac{1}{4} & \vdots & \frac{1}{8} \\ 0 & 0 & 1 & \vdots & \frac{1}{6} \end{bmatrix}} \rightarrow \underset{\frac{5}{2}(R_3) + R_1 \rightarrow R_1}{\begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{2}{3} \\ 0 & 1 & -\frac{1}{4} & \vdots & \frac{1}{8} \\ 0 & 0 & 1 & \vdots & \frac{1}{6} \end{bmatrix}} \rightarrow \underset{\frac{1}{4}(R_3) + R_2 \rightarrow R_2}{\begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{2}{3} \\ 0 & 1 & 0 & \vdots & \frac{1}{6} \\ 0 & 0 & 1 & \vdots & \frac{1}{6} \end{bmatrix}}$$

$p_1 = \frac{2}{3}, p_2 = \frac{1}{6}, p_3 = \frac{1}{6}$