

## Chapter 3 Parent Guide

### Systems of Linear Equations and Inequalities

**To continue in this course, students need** to master solving equations by using the most expeditious technique. These skills will be applied to nonlinear functions throughout the rest of this course. Students who are not skilled in choosing the best technique for solving can get bogged down in solving just one equation. Once they master all the different solving techniques, they will understand which problem-solving method works best in solving certain equations.

Chapter 3 reviews solving systems of linear equations. Each of the first two lessons in the chapter involves one or two different solving techniques. The last four lessons apply that information to solve systems of linear inequalities, to solve problems involving linear programming, and to solve parametric equations.

Linear programming is graphing two or more inequalities to find the optimal solution to a problem involving several constraints. Parametric equations are a special type of system of equations.

Lesson 3.1 reviews solving a system of linear equations by graphing and by substitution. Lesson 3.2 involves the solving technique of elimination, or solving by adding or subtracting equations to eliminate a variable. Lessons 3.3 and 3.4 involve solving linear inequalities in two variables and solving systems of inequalities. Lesson 3.5 addresses linear programming. Lesson 3.6 discusses parametric equations.

Work the following activity with your son or daughter to show how systems of inequalities can be used to solve complex problems involving several parameters.

### PROBLEM FOR DISCUSSION (See textbook page 179)

Suppose that an actor is needed for the lead character in a play. The search is on for a male actor between the ages of 25 and 35, standing between 5' 8" and 5' 10" tall, with a medium build, and within the weight ranges given in the table for males with medium builds. Do you know someone who fits this description?

1. Name the various restrictions for the actor.

The actor needs to be older than 25, but younger than 35. You can say that his age,  $a$ , needs to be greater than or equal to 25 but less than or equal to 35. In mathematics, this is written as,  $a \geq 25$  and  $a \leq 35$ .

This can also be written more compactly as,  $25 \leq a \leq 35$ .

The actor needs to be 5'8" or taller, but 5'10" or shorter. Converting these measurements to inches,  $5'8" = 12(5) + 8 = 68"$  and  $5'10" = 5(12) + 10 = 70"$  tall.

Using the table from the book to determine the actor's weight, you see that for an actor 68" tall, the least he can weigh is 145 pounds and the most he can weigh is 157 pounds. For an actor 70" tall, the least he can weigh is 151 pounds and the most he can weigh is 163 pounds.

2. Look at the system of linear inequalities to the left of the graph. Which restriction does each represent?

The first inequality,  $68 \leq h \leq 70$ , is the restriction on the actor's height.

The second inequality,  $w \geq 3h - 59$ , represents the line determined by the following set of points:

$\{(66, 139), (67, 142), (68, 145), (69, 148), (70, 151)\}$ .

In this set, the  $x$ -coordinates are the possible heights and the  $y$ -coordinates are the "lower-end" weights.

The third inequality,  $w \leq 3h - 47$ , represents the line determined by the following set of points:

$\{(66, 151), (67, 154), (68, 157), (69, 160), (70, 163)\}$ .

In this set, the  $x$ -coordinates are the possible heights and the  $y$ -coordinates are the "upper-end" weights.

3. Look at the graph of the system of linear inequalities. What does the blue region represent? What do the red lines represent?

The blue region represents the solutions to the inequalities. This means that all of the coordinate pairs that make the inequalities true lie in these regions.

The red lines represent the graphs of the lines

$w = 3h - 59$ ,  $w = 3h - 47$ ,  $h = 68$ , and  $h = 70$ .

4. Discuss why there are three inequalities and four lines on the graph.

There are four lines because  $68 \leq h \leq 70$  can be written as  $h \geq 68$  and  $h \leq 70$ . The other two inequalities are  $w \leq 3h - 47$  and  $w \geq 3h - 59$ .

5. Does the graph show a region where all restrictions are met? How can you tell?

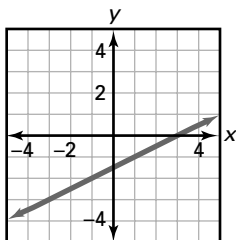
Yes. All of the regions meet in the darker blue region. This is the region where all of the sets of solutions intersect.

The following are complete worked out solutions to selected exercises in the student textbook. These solutions are provided to you so that you can help your child with their homework. Your child's classroom notes, example problems in the text, and these worked out solutions are all useful tools to help you and your child work through their assignment.

## Lesson 3.1

11. The system is dependent. There are infinitely many solutions.

20.



The system is dependent. There are infinitely many solutions.

$$36. \begin{cases} 2x + y = -9 \\ 3x + y = 11 \end{cases}$$

Solve for  $y$  in terms of  $x$  in the first equation.

$$2x + y = -9$$

$$y = -2x - 9$$

Substitute  $-2x - 9$  for  $y$  in the second equation then solve for  $x$ .

$$3x + y = 11$$

$$3x + (-2x - 9) = 11$$

$$3x - 2x - 9 = 11$$

$$x = 20$$

Substitute 20 for  $x$  in either original equation to find  $y$ .

$$2x + y = -9$$

$$2(20) + y = -9$$

$$40 + y = -9$$

$$y = -49$$

The solution is  $(20, -49)$ .

$$42. \begin{cases} 2x - 3y + 4z = 8 \\ 3x + 2y = 7 \\ x = 1 \end{cases}$$

Substitute 1 for  $x$  in the second equation to solve for  $y$ .

$$3x + 2y = 7$$

$$3(1) + 2y = 7$$

$$2y = 4$$

$$y = 2$$

Substitute 1 for  $x$  and 2 for  $y$  in the first equation to find  $z$ .

$$2x - 3y + 4z = 8$$

$$2(1) - 3(2) + 4z = 8$$

$$4z = 12$$

$$z = 3$$

The solution is  $(1, 2, 3)$ .

## Chapter 3

- 58.** Let the number of milliliters used of the 1% solution be  $x$  and let  $y$  be the number of milliliters used of the 10% solution.

Since there will be 90 milliliters total,  $x + y = 90$ .

And

$$\begin{aligned}1\% \times x + 10\% \times y &= 3\% \times 90 \\0.01x + 0.10y &= 0.03 \times 90 \\0.01x + 0.10y &= 2.7\end{aligned}$$

Thus, we need to solve the system of equations

$$\begin{cases}x + y = 90 \\0.01x + 0.10y = 2.7\end{cases}$$

Solve the first equation for  $y$  in terms of  $x$

$$\begin{aligned}x + y &= 90 \\y &= 90 - x\end{aligned}$$

Substitute  $90 - x$  for  $y$  in the second equation and solve for  $x$ .

$$\begin{aligned}0.01x + 0.10y &= 2.7 \\0.01x + 0.10(90 - x) &= 2.7 \\0.01x + 9 - 0.10x &= 2.7 \\-0.09x &= -6.3 \\x &= 70\end{aligned}$$

Substitute 70 for  $x$  in the first equation and solve for  $y$ .

$$\begin{aligned}x + y &= 90 \\70 + y &= 90 \\y &= 20\end{aligned}$$

The students should use 70 milliliters of the 1% acid solution and 20 milliliters of the 10% solution.

## Lesson 3.2

**13.** 
$$\begin{cases}2s - 5t = 22 \\2s - 3t = 6\end{cases}$$

To eliminate  $s$ , multiply the first equation by  $-1$  and combine the resulting equations.

$$\begin{aligned}\begin{cases}2s - 5t = 22 \\2s - 3t = 6\end{cases} &\rightarrow \begin{cases}(-1)(2s - 5t) = (-1)(22) \\2s - 3t = 6\end{cases} \rightarrow \begin{cases}-2s + 5t = -22 \\2s - 3t = 6\end{cases} \\-2s + 5t &= -22 \\ \underline{2s - 3t} &= \underline{6} \\2t &= -16 \\t &= -8\end{aligned}$$

Substitute  $-8$  for  $t$  in either original equation to find  $s$ .

$$\begin{aligned}2s - 5t &= 22 \\2s - 5(-8) &= 22 \\2s &= -18 \\s &= -9\end{aligned}$$

The solution is  $s = -9$  and  $t = -8$ , or  $(-9, -8)$ .

## Chapter 3

$$33. \begin{cases} -2x + 5y = -23 \\ 24 + 4y = 3x \end{cases}$$

Arrange the second equation in standard form.

$$\begin{cases} -2x + 5y = -23 \\ 24 + 4y = 3x \end{cases} \rightarrow \begin{cases} -2x + 5y = -23 \\ -3x + 4y = -24 \end{cases}$$

To eliminate  $x$ , multiply the first equation by 3 and the second equation by  $-2$  and combine the resulting equations.

$$\begin{cases} -2x + 5y = -23 \\ -3x + 4y = -24 \end{cases} \rightarrow \begin{cases} 3(-2x + 5y) = 3(-23) \\ (-2)(-3x + 4y) = (-2)(-24) \end{cases} \rightarrow \begin{cases} -6x + 15y = -69 \\ 6x - 8y = 48 \end{cases}$$

$$\begin{array}{r} -6x + 15y = -69 \\ \underline{6x - 8y = 48} \\ 7y = -21 \\ y = -3 \end{array}$$

Substitute  $-3$  for  $y$  in either original equation to find  $x$ .

$$\begin{aligned} -2x + 5y &= -23 \\ -2x + 5(-3) &= -23 \\ -2x - 15 &= -23 \\ -2x &= -8 \\ x &= 4 \end{aligned}$$

The solution is  $x = 4$  and  $y = -3$ , or  $(4, -3)$ .

43. Let  $x$  be the number of light packages and  $y$  be the number of heavy packages.

	Packages less than 3lb	Packages at least 3 lb	Total
Number of packages	$x$	$y$	12
Shipping and Handling Charge	$2x$	$3y$	29

We obtain the system  $\begin{cases} x + y = 12 \\ 2x + 3y = 29 \end{cases}$

To eliminate  $x$ , multiply the first equation by  $-2$ .

$$\begin{cases} x + y = 12 \\ 2x + 3y = 29 \end{cases} \rightarrow \begin{cases} -2x - 2y = -24 \\ 2x + 3y = 29 \end{cases}$$

Combine the resulting equations and solve for  $y$ .

$$\begin{array}{r} -2x - 2y = -24 \\ \underline{2x + 3y = 29} \\ y = 5 \end{array}$$

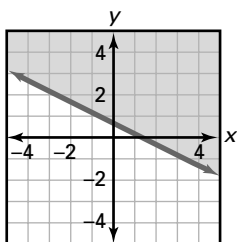
Substitute 5 for  $y$  in the first original equation to find  $x$ .

$$\begin{aligned} x + y &= 12 \\ x + 5 &= 12 \\ x &= 7 \end{aligned}$$

The company shipped 7 packages under 3 lb and 5 packages weighing 3 lb or more.

## Lesson 3.3

15.  $y \geq -\frac{1}{2}x + \frac{2}{3}$



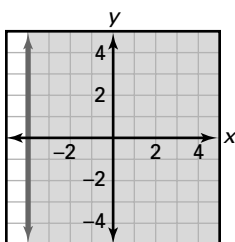
Graph the boundary line  $y = -\frac{1}{2}x + \frac{2}{3}$ ;

use a solid line. Use  $(0, 0)$  as a test point:

$$0 \geq -\frac{1}{2}(0) + \frac{2}{3} \text{ is false, so shade the region}$$

that doesn't contain  $(0, 0)$ .

38.  $-x \leq 4$



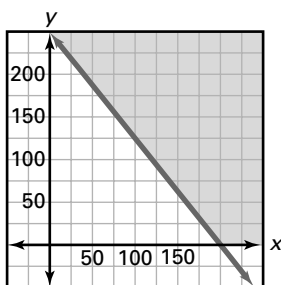
Graph the boundary line,  $x = -4$ ; use a solid line.

Since  $-x \leq 4$  is equivalent to  $x \geq -4$ , shade the region to the right of the line.

52. Let  $x$  be the number of soft drinks sold and  $y$  be the number of ice-cream bars sold. The profit from sales of  $x$  soft drinks and  $y$  ice-cream bars is then  $0.25x + 0.20y$ .

a.  $0.25x + 0.20y \geq 50$

b.



- c. For a profit of exactly \$50, choose any point on the boundary line— $(200, 0)$ ,  $(0, 250)$ ,  $(100, 125)$ ,  $(4, 245)$ , for example.
- d. For a profit of more than \$50, choose any point in the shaded region— $(100, 150)$ ,  $(100, 175)$ ,  $(100, 200)$ , for example.
- e. For a profit of less than \$50, choose any point with  $x \geq 0$ ,  $y \geq 0$ , and not in the shaded region— $(100, 25)$ ,  $(100, 50)$ ,  $(100, 75)$ , for example.

## Chapter 3

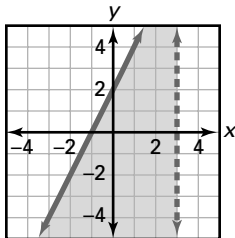
### Lesson 3.4

10. 
$$\begin{cases} x < 3 \\ y \leq 2x + 2 \end{cases}$$

Graph  $x < 3$ , the region lying to the left of the dashed vertical boundary line,  $x = 3$ .

Graph  $y \leq 2x + 2$ , the region lying below the solid boundary line,  $y = 2x + 2$ .

The solution is the intersection of the graphed regions.



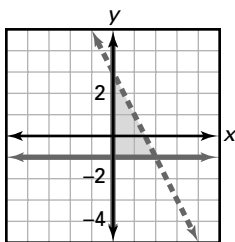
16. 
$$\begin{cases} x \geq 0 \\ y \geq -1 \\ y < -2x + 3 \end{cases}$$

Graph  $x \geq 0$ , the region to the right of the solid vertical boundary line,  $x = 0$ .

Graph  $y \geq -1$ , the region lying above the solid horizontal boundary line,  $y = -1$ .

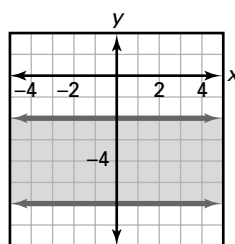
Graph  $y < -2x + 3$ , the region lying below the dashed boundary line,  $y = -2x + 3$ .

The solution is the intersection of the graphed regions.



26.  $-6 \leq y \leq -2$

The solution is the region lying between the two solid horizontal boundary lines,  $y = -6$  and  $y = -2$ .



## Chapter 3

- 35.** First find equations for the boundary lines. The first line is through the points  $(-3, -3)$  and  $(0, 3)$ .

$$m = \frac{3 - (-3)}{0 - (-3)} = \frac{6}{3} = 2$$

The line has  $y$ -intercept 3, so it has equation  $y = 2x + 3$ . The second line is through the points  $(0, 3)$  and  $(1, 0)$ .

$$m = \frac{0 - 3}{1 - 0} = -3$$

This line has  $y$ -intercept 3, so it has equation  $y = -3x + 3$ . The third line is the vertical line through  $x = 1$ , so it has equation  $x = 1$ . The fourth line is the horizontal line through  $y = -3$ , so it has equation  $y = -3$ . Now give each boundary line the appropriate inequality symbol. Because the first, third, and fourth lines are solid, use  $\leq$  or  $\geq$ . The second line is dashed, so use  $<$  or  $>$ .

below the first line:  $y \leq 2x + 3$

below the second line:  $y < -3x + 3$

left of the third line:  $x \leq 1$

above the fourth line:  $y \geq -3$

The system of inequalities is: 
$$\begin{cases} y \leq 2x + 3 \\ y < -3x + 3 \\ x \leq 1 \\ y \geq -3 \end{cases}$$

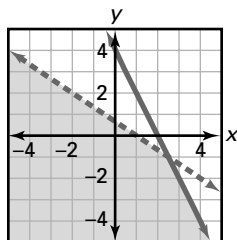
**39.** 
$$\begin{cases} x + \frac{1}{2}y \leq 2 \\ 2x + 3y < 2 \end{cases}$$

Graph  $x + \frac{1}{2}y \leq 2$ , the region below the solid boundary line,  $y = -2x + 4$ .

Graph  $2x + 3y < 2$ , the region lying below the dashed boundary line,

$$y = -\frac{2}{3}x + \frac{2}{3}$$

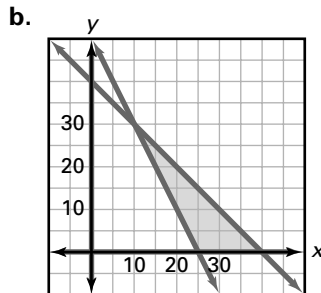
The solution is the intersection of the graphed regions.



## Chapter 3

50. a. Let  $x$  represent hours spent programming, and  $y$  represent hours spent tutoring.

$$\begin{cases} x + y \leq 40 & \text{hours worked} \\ 20x + 10y \geq 500 & \text{money made} \\ x \geq 0 \\ y \geq 0 \end{cases}$$

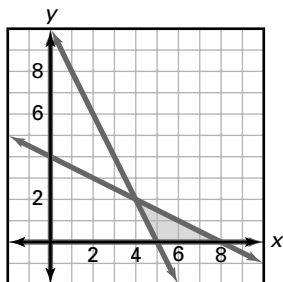


Yes it is a polygon.

- c. Answers may vary. Sample answer:  $(25, 10)$  is in the shaded region and is interpreted as 25 hours per week of programming and 10 hours per week tutoring.
- d. Perhaps the best choice would be  $x = 40, y = 0$ —Angela should spend all of her hours programming—since for a total of 40 hours per week, it yields the maximum earnings of  $20(40) + 10(0) = 800$  dollars.

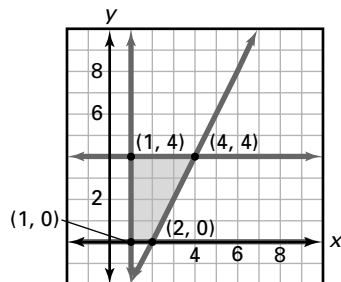
## Lesson 3.5

10. 
$$\begin{cases} x + 2y \leq 8 \\ 2x + y \geq 10 \\ x \geq 0, y \geq 0 \end{cases}$$



17. There is only one vertex to identify:  $(0, 1)$ .

25. 
$$\begin{cases} 4x - 2y \leq 8 \\ x \geq 1 \\ 0 \leq y \leq 4 \end{cases}$$



Vertex	Objective function
$(1, 0)$	$P = 2(1) + 7(0) = 2$
$(1, 4)$	$P = 2(1) + 7(4) = 30$
$(4, 4)$	$P = 2(4) + 7(4) = 36$
$(2, 0)$	$P = 2(2) + 7(0) = 4$

The maximum value, 36, of  $P = 2x + 7y$  occurs at  $(4, 4)$ .

The minimum value, 2,  $P = 2x + 7y$  occurs at  $(1, 0)$ .

## Chapter 3

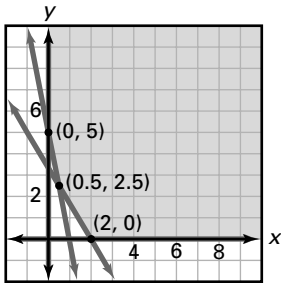
35. a. The data can be represented by the table.

	meat	vegetables	available
protein	$45x$	$9y$	45
iron	$10x$	$6y$	20
fat	$4x$	$2y$	

We obtain the constraints

$$\begin{cases} 45x + 9y \geq 45 \\ 10x + 6y \geq 20 \\ x \geq 0, y \geq 0 \end{cases}$$

- b.



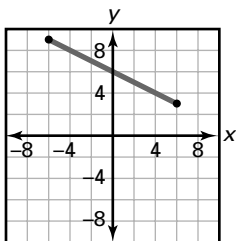
- c. From our table, the objective function is  $M = 4x + 2y$ .

Vertex	Objective function
(2, 0)	$M = 4(2) + 2(0) = 8$
(0.5, 2.5)	$M = 4(0.5) + 2(2.5) = 7$
(0, 5)	$M = 4(0) + 2(5) = 10$

The minimum number of grams of fat is seven and occurs when one-half of a 3-ounce serving of meat and 2.5 1-cup servings of vegetables are served.

## Lesson 3.6

11.  $\begin{cases} x(t) = 2t \\ y(t) = 6 - t \end{cases}$  for  $-3 \leq t \leq 3$



17.  $\begin{cases} x(t) = 2t + 1 \\ y(t) = t + 5 \end{cases}$

$$x(t) = 2t + 1 \rightarrow x = 2t + 1$$

$$x - 1 = 2t$$

$$t = \frac{1}{2}(x - 1)$$

$$y(t) = t + 5 \rightarrow y = t + 5$$

$$y = \frac{1}{2}(x - 1) + 5$$

$$y = \frac{1}{2}x + \frac{9}{2}$$

$$\text{So, } y = \frac{1}{2}x + \frac{9}{2}.$$

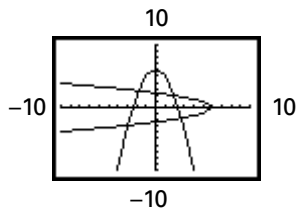
## Chapter 3

29. original function:

$$\begin{cases} x(t) = t \\ y(t) = 6 - t^2 \end{cases}$$

inverse function:

$$\begin{cases} x(t) = 6 - t^2 \\ y(t) = t \end{cases}$$



36. 
$$\begin{cases} x(t) = 8.2t \\ y(t) = 2 + 5.7t - 4.9t^2 \end{cases}$$

- Use the trace feature to find the height of the ball at the net, or at a horizontal distance of 9 m. We find  $y(t) \approx 2.34$  when  $t \approx 1.1$ . That is, the ball reaches an altitude of 2.34 meters, and  $2.34 - 2.2 = 0.14$  meters above the net.
- Using the calculator's trace feature to find when the altitude is zero, we find  $x(t) \approx 11.8$  when  $t \approx 1.44$ .  
The ball travels 11.8 meters.
- $9 \leq x \leq 18$   
The ball must land within 9 meters of the net which is 9 meters away.  
From part **a.**, the ball travels 11.89 meters, so it does land where it should.