

Chapter 2 Parent Guide Numbers and Functions

Chapter 2 reviews sets of numbers, operations with numbers, exponents, and functions. The second half of the chapter builds into operations with functions, inverses of functions, piecewise functions, step functions, and absolute value functions. The graphing lesson examines transformations of graphs.

Familiarizing the student with these functions will benefit the student who plans to take calculus. It will also enable students to recognize and interpret graphs that can appear in business reports.

In Lesson 2.1, your son or daughter will identify and use properties of real numbers and evaluate expressions by using the order of operations. In Lesson 2.2, students evaluate and simplify expressions involving exponents.

Lesson 2.3 reviews graphing a relation and using function notation. Lesson 2.4 involves operations with functions and composition of functions. In Lesson 2.5, students find the inverse of a relation or function and determine if the inverse is itself a function. Lesson 2.6 discusses piecewise, step, and absolute-value functions. Lesson 2.7 examines translations of graphs.

By the end of this chapter, students should be comfortable with function notation. It will be used often throughout the rest of this course and future courses in mathematics and physics.

Your child has found the inverse of a function in Algebra 1, except the process was not named. Go through the following activity with your son or daughter to help him or her recall what was learned in Algebra 1.

PROBLEM FOR DISCUSSION (See textbook page 118)

Many real-world relationships can be represented with a pair of inverse functions. For example, the equations used to convert degrees Fahrenheit to degrees Celsius and vice versa are inverse functions. In Lesson 1.6, you used the equation $F = \frac{9}{5}C + 32$ to find the Celsius temperature corresponding to 86°F . How can you find the Celsius temperatures that correspond to Fahrenheit temperatures?

1. Discuss the graph of $F = \frac{9}{5}C + 32$. Find the slope and y-intercept.

This equation is an example of a linear relationship. If you use variables y and x in place of F and C , you can easily see how the equation is written in slope-intercept form, $y = mx + b$.

$$F = \frac{9}{5}C + 32$$

$$y = \frac{9}{5}x + 32$$

In the form, $y = mx + b$, m is the slope of the line and b is the y-intercept. So, for this equation, $\frac{9}{5}$ is the slope and 32 is the

y-intercept.

2. Discuss why $F = \frac{9}{5}C + 32$ is a function.

The definition of a function is a relation in which, for each x value, there is exactly *one* corresponding y value. The vertical line test is used to determine if a relation is a function. If the vertical line intersects the graphed relation more than once, then the relation is not a function.

If you pass a vertical line across the graph of $F = \frac{9}{5}C + 32$, it does not cross more than once at any point. Therefore, the relation is a function.

3. Find the inverse of $F = \frac{9}{5}C + 32$ by rewriting it as a function of C in terms of F .

To find the inverse of $F = \frac{9}{5}C + 32$, solve the equation for C .

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C + 32 - 32 \quad \text{Subtract 32 from both sides of the equation.}$$

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9}(F - 32) = \frac{5}{9}\left(\frac{9}{5}C\right) \quad \text{Multiply both sides by } \frac{5}{9}, \text{ the reciprocal of } \frac{9}{5}.$$

$$\frac{5}{9}(F - 32) = C$$

$$\frac{5}{9}F - \frac{160}{9} = C$$

4. Find the slope and y-intercept of the inverse of $F = \frac{9}{5}C + 32$.

This is a linear relationship that can be written with x and y , as follows: $\frac{5}{9}x - \frac{160}{9} = y$. Using slope-intercept form, $y = mx + b$,

where m is the slope and b is the y-intercept, $\frac{5}{9}$ is the slope and $\frac{160}{9}$ is the y-intercept.

Chapter 2

74. $71.33^{0.44} + 478.2^{0.4} \approx 18.3$

81. a. $R = \frac{2}{3}lr^{-4}$
 $= \frac{2}{3}(0.2)(0.015)^{-4}$
 $\approx 2,633,744.9$

b. $R = \frac{2}{3}lr^{-4}$
 $= \frac{2}{3}l\frac{1}{r^4}$
 $= \frac{2l}{3r^4}$

Lesson 2.3

19. For each first coordinate, there is exactly one second coordinate. The relation represents a function.

35. The graph represents a function. Every vertical line intersects the graph at no more than one point.

46. $g(x) = \frac{x-4}{5}$

$$g(-9) = \frac{(-9)-4}{5}$$

$$g(-9) = -\frac{13}{5}$$

$$g(9) = \frac{(9)-4}{5}$$

$$g(9) = 1$$

67. Let S be the sale price and p be the original price. Then $S = p - \text{discount}$.

If the discount is 30% or 0.3 of the original price, then

$$S(p) = p - 0.3p = (1 - 0.3)p = 0.7p$$

Lesson 2.4

12. $f + g = f(x) + g(x)$
 $= (20x + 7) + (-3x)$

$$f + g = 17x + 7$$

$$f - g = f(x) - g(x)$$

$$= (20x + 7) - (-3x)$$

$$f - g = 23x + 7$$

22. $f \cdot g = f(x) \cdot g(x)$
 $= (7x^2)(2x - 6)$

$$f \cdot g = 14x^3 - 42x^2$$

$$\frac{f}{g} = \frac{f(x)}{g(x)}, g(x) \neq 0$$

$$\frac{f}{g} = \frac{7x^2}{2x-6}, x \neq 3$$

Chapter 2

26. $f - g = f(x) - g(x)$ Definition of $f - g$
 $= (x^2 - 1) - (2x - 3)$ Substitution
 $f - g = x^2 - 2x + 2$ Distributive Property/Combine like terms

33. $f \cdot g = f(x) \cdot g(x)$ Definition of $f \cdot g$
 $= (x - 3)(x^2 - 9)$ Substitution
 $= x(x^2 - 9) - 3(x^2 - 9)$ Distributive Property
 $= x^3 - 9x - 3x^2 + 27$ Distributive Property
 $f \cdot g = x^3 - 3x^2 - 9x + 27$ Commutative Property

41. $f \circ g = f(g(x))$
 $= f(3)$
 $= -4(3)^2 + 3(3) - 1$
 $= -36 + 9 - 1$
 $f \circ g = -28$
 $g \circ f = g(f(x))$
 $= g(-4x^2 + 3x - 1)$
 $g \circ f = 3$

61. a. $C = C(x(t)) = C(90t) = 4(90t) + 850 = 360t + 850.$

b. $C(5) = 360(5) + 850 = 2650$

After 5 hours, the cost of production is \$2,650.

c. $x(5) = 90(5) = 450$

450 picture frames are produced in 5 hours.

Lesson 2.5

14. Relation: $\{(-1, -6), (0, 2), (1, 2), (3, 6)\}$
Inverse: $\{(-6, -1), (2, 0), (2, 1), (6, 3)\}$

The given relation is a function because each domain member is paired with exactly one range member. The inverse is not a function because 2 is paired with 0 and 1.

23. Function: $\{(5, 2), (4, 3), (3, 5), (2, 3)\}$
Inverse: $\{(2, 5), (3, 4), (5, 3), (3, 2)\}$
The inverse is not a function because 3 is matched with 4 and 2.

Chapter 2

32. $g(x) = \frac{x-1}{4}$

In $y = \frac{x-1}{4}$, interchange x and y .

Then solve for y .

$$x = \frac{y-1}{4}$$

$$4x = y - 1$$

$$y = 4x + 1$$

So, $g^{-1}(x) = 4x + 1$.

Show $g^{-1}(g(x)) = x$.

$$\begin{aligned} g^{-1}(g(x)) &= g^{-1}\left(\frac{x-1}{4}\right) \\ &= 4\left(\frac{x-1}{4}\right) + 1 \\ &= (x-1) + 1 \\ &= x \end{aligned}$$

Now show $g(g^{-1}(x)) = x$.

$$\begin{aligned} g(4x + 1) &= \frac{(4x + 1) - 1}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

36. $f(x) = \frac{1}{3}x - 1$

In $y = \frac{1}{3}x - 1$, interchange x and y .

Then solve for y .

$$x = \frac{1}{3}y - 1$$

$$x + 1 = \frac{1}{3}y$$

$$y = 3(x + 1)$$

So, $f^{-1}(x) = 3x + 3$.

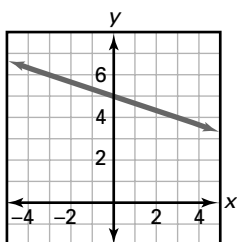
Show $f^{-1}(f(x)) = x$.

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}\left(\frac{1}{3}x - 1\right) \\ &= 3\left(\frac{1}{3}x - 1\right) + 3 \\ &= (x - 3) + 3 \\ &= x \end{aligned}$$

Now show $f(f^{-1}(x)) = x$.

$$\begin{aligned} f(f^{-1}(x)) &= f(3x + 3) \\ &= \frac{1}{3}(3x + 3) - 1 \\ &= (x + 1) - 1 \\ &= x \end{aligned}$$

42.



By the horizontal line test, the inverse is a function.

Chapter 2

52. a. $p = 84x + 60,000$

b. In $p = 84x + 60,000$, solve for x .

$$p - 60,000 = 84x$$

$$x = \frac{p - 60,000}{84}$$

x represents the size of a house that costs p dollars.

c. If $p = 180,000$:

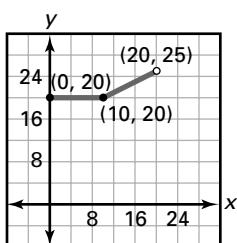
$$x = \frac{180,000 - 60,000}{84}$$

$$x \approx 1429$$

That is, a house of approximately 1429 square feet may be purchased for \$180,000.

Lesson 2.6

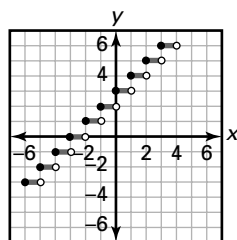
18.



$$26. f(x) = \begin{cases} -x - 2 & \text{if } x < -2 & x \leq -2 \\ \frac{4}{3}x + \frac{8}{3} & \text{if } -2 \leq x < 1 \text{ or } -2 < x \leq 1 \\ -7x + 11 & \text{if } 1 \leq x < 2 & 1 < x \leq 2 \\ -3 & \text{if } x \geq 2 & x > 2 \end{cases}$$

$$32. [-4.1] - [-3.25] \\ = -5 - (-4) \\ = -1$$

54.



$$63. |xy| = |x| \cdot |y|$$

True for all values of x and y .

Chapter 2

74. a.

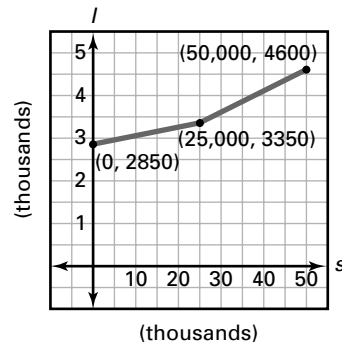
monthly sales	monthly income
5,000	2950
10,000	3050
15,000	3150
20,000	3250
25,000	3350
30,000	3600
35,000	3850
40,000	4100
45,000	4350
50,000	4600

c. $0 < s \leq 25,000$, the income is given by $2850 + 0.02s$. For $s > 25,000$, the income is given by

$$2850 + 0.02 \cdot 25,000 + 0.05(s - 25,000) = 0.05s - 2100. \text{ Or,}$$

$$I(s) = \begin{cases} 2850 + 0.02s & \text{if } 0 \leq s \leq 25,000 \\ 0.05s + 2100 & \text{if } 25,000 < s \leq 50,000 \end{cases}$$

b.



d. $I(43,000) = 0.05(43,000) + 2100 = 4250$
The salesperson's income for \$43,000 in sales is \$4,250.

Lesson 2.7

13. $g(x) = f(4x) = (4x)^2$ is a horizontal compression by a factor of $\frac{1}{4}$.

23. $g(x) = \frac{1}{3}f(x) - 1 = \frac{1}{3}x^2 - 1$ is a vertical compression by a factor of $\frac{1}{3}$ followed by a vertical translation of 1 unit down.

31. $g(x) = f(-4x) = \sqrt{-4x}$ is a horizontal compression by a factor of $\frac{1}{4}$ and a horizontal reflection across the y -axis.

46. $g(x) = 3f(x) = 3x^2$

51. $g(x) = f(-x) = 2(-x) - 1$

62.

