

Section Overview

Greatest Common Factors and Factoring by GCF

Lessons 8-1, 8-2

Why? The first step in factoring any polynomial is to look for the greatest common factor of the terms.

Factors that are shared by two or more whole numbers are called **common factors**. The greatest is called the **greatest common factor**, or **GCF**.

$$8x^4 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x$$

$$12x^3 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$$

$2 \cdot 2 \cdot x \cdot x \cdot x$
 The GCF of $8x^4$ and $12x^3$ is $4x^3$.

A greatest common factor can be a constant or a polynomial.

Monomial GCF

$$18y^6 + 6y^2 - 9y$$

$$3y(6y^5 + 2y - 3)$$

Binomial GCF

$$5x^2(x - 3) + 4(x - 3)$$

$$(x - 3)(5x^2 + 4)$$

When a polynomial has four terms, you can make two groups and look for a common binomial. This is called *factoring by grouping*.

$$a^3 + 3a + 7a^2 + 21$$

$$(a^3 + 3a) + (7a^2 + 21)$$

$$a(a^2 + 3) + 7(a^2 + 3)$$

$$(a^2 + 3)(a + 7)$$

Factoring $x^2 + bx + c$ and $ax^2 + bx + c$

Lessons 8-3, 8-4

Why? Factoring a trinomial will help students determine where a parabola intersects the x-axis.

Factoring $x^2 + bx + c$

To factor $x^2 + bx + c$, find two factors of c whose sum is b .

If c is positive: $x^2 + 12x + 20$

Factors of 20	Sum
1 and 20	21 ✗
2 and 10	12 ✓
4 and 5	9 ✗

$(x + 2)(x + 10)$

If c is negative: $x^2 + 3x - 18$

Factors of -18	Sum
-1 and 18	17 ✗
-2 and 9	7 ✗
-3 and 6	3 ✓

$(x - 3)(x + 6)$

Factoring $ax^2 + bx + c$

To factor $ax^2 + bx + c$, find factors of a and factors of c . The sum of the outer and inner products should be b .

If c is positive: $3x^2 + 17x + 10$

Factors of 3	Factors of 10	Outer + Inner
$1 \cdot 3$	$1 \cdot 10$	$1 \cdot 10 + 3 \cdot 1 = 13$ ✗
$1 \cdot 3$	$10 \cdot 1$	$1 \cdot 1 + 3 \cdot 10 = 31$ ✗
$1 \cdot 3$	$2 \cdot 5$	$1 \cdot 5 + 3 \cdot 2 = 11$ ✗
$1 \cdot 3$	$5 \cdot 2$	$1 \cdot 2 + 3 \cdot 5 = 17$ ✓

$(1x + 5)(3x + 2) = (x + 5)(3x + 2)$

If c is negative: $5x^2 - 13x - 6$

Factors of 5	Factors of -6	Outer + Inner
$1 \cdot 5$	$1(-6)$	$1(-6) + 5(1) = -1$ ✗
$1 \cdot 5$	$6(-1)$	$1(-1) + 5(6) = 29$ ✗
$1 \cdot 5$	$3(-2)$	$1(-2) + 5(3) = 13$ ✗
$1 \cdot 5$	$-3(2)$	$1(2) + 5(-3) = -13$ ✓

$(1x - 3)(5x + 2) = (x - 3)(5x + 2)$