Simplifying Expressions (for Holt Algebra 1, Lesson 1-7)

- The Commutative and Associative Properties of Addition and Multiplication allow you to rearrange and simplify an expression.
- The Distributive Property can be used with addition or subtraction. It is often used as a mental math strategy.

Properties of Addition and Multiplication

PROPERTY	WORDS	NUMBERS	ALGEBRA
Commutative	You can add numbers in any order and multiply numbers in any order.	$2 + 7 = 7 + 2$ $3 \bullet 9 = 9 \bullet 3$	$a + b = b + a$ $ab = ba$
Associative	When you are only adding or multiplying, you can group any of the numbers together.	$6 + 8 + 2 = (6 + 8) + 2 = 6 + (8 + 2)$ $7 \bullet 4 \bullet 5 = (7 \bullet 4) \bullet 5 = 7 \bullet (4 \bullet 5)$	$a + b + c = (a + b) + c = a + (b + c)$ $abc = (ab)c = a(bc)$
Distributive	You can multiply a number by a sum or multiply by each number in the sum and then add. The result is the same.	$3(4 + 8) = 3(4) + 3(8)$	$a(b + c) = a(b) + a(c)$

- The **like terms** of an expression contain the same variables raised to the same powers. You can combine like terms by adding or subtracting the **coefficients** (the numbers multiplying the variables) and keeping the variables and exponents the same.

Examples

USING THE COMMUTATIVE, ASSOCIATIVE, AND DISTRIBUTIVE PROPERTIES

1. Simplify each expression.

A $4 \bullet 9 \bullet 25$

$9 \bullet 4 \bullet 25$ *Commutative Property*

$9 \bullet (4 \bullet 25)$ *Associative Property*

$9 \bullet 100$ *Simplify parentheses.*

900

B $15(103)$

$15(100 + 3)$ *Rewrite 103 as 100 + 3.*

$15(100) + 15(3)$ *Distributive Property*

$1500 + 45$ *Multiply.*

1545 *Add.*

COMBINING LIKE TERMS

2. Simplify each expression by combining like terms.

A $12x + 30x$

These are like terms. Add the coefficients.

$12x + 30x = 42x$

B $8y^2 - y^2$

These are like terms. y^2 has a coefficient of 1.

$8y^2 - 1y^2 = 7y^2$

C $4n + 11n^2$

These are not like terms. Do not combine them.

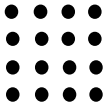
$4n + 11n^2$

Powers and Exponents (for Holt Algebra 1, Lesson 1-4)

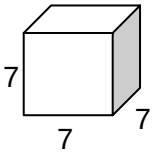
- A **power** is an expression written with an *exponent* and a *base*.
- The **base** is the number used as a factor.
- The **exponent** tells how many times the base is used as a factor.

base → 5^3 ← exponent $\qquad 5^3 = 5 \bullet 5 \bullet 5 = 125$

- Powers of 2 and 3 can be represented by geometric models.



4^2



7^3

- Powers can be written and evaluated using repeated multiplication.

Reading Exponents			
Words	**Multiplication**	**Power**	**Value**
3 to the first power	3	3^1	3
3 to the second power, or 3 squared	3 • 3	3^2	9
3 to the third power, or 3 cubed	3 • 3 • 3	3^3	27
3 to the fourth power	3 • 3 • 3 • 3	3^4	81
3 to the fifth power	3 • 3 • 3 • 3 • 3	3^5	243

Examples

EVALUATING POWERS

1. Evaluate each expression.

A $(-2)^3$

Use –2 as a factor 3 times.

$(-2)(-2)(-2) = -8$

B -5^2

Think of a negative in front of a power as –1.

$-1 \bullet 5 \bullet 5 = -25$

C $\left(\frac{2}{3}\right)^2$

Use $\frac{2}{3}$ as a factor 2 times.

$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

WRITING POWERS

2. Write each number as the power of a given base.

A 8; base 2

The product of three 2's is 8.

$2 \bullet 2 \bullet 2 = 2^3$

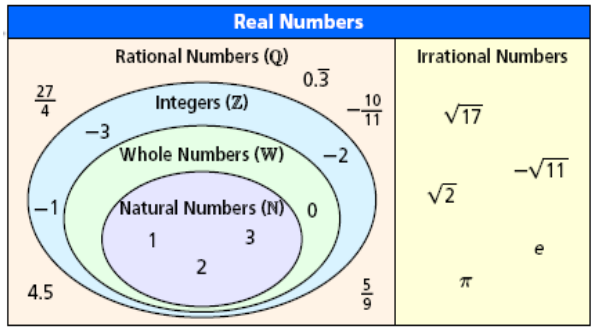
B –125; base –5

The product of three –5's is –125.

$(-5)(-5)(-5) = (-5)^3$

Square Roots and Real Numbers (for Holt Algebra 1, Lesson 1-5)

- A number that is multiplied by itself to form a product is called a **square root** of that product.
- A **perfect square** is a number whose positive square root is a *whole number*. Some perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.
- **Real numbers** can be represented on the number line. They can be classified according to their characteristics, as follows:
- The **natural numbers** are the counting numbers (1, 2, 3...); the **whole numbers** are the natural numbers and 0.
- **Integers** are whole numbers and their opposites (...–3, –2, –1, 0, 1, 2, 3...).
- **Rational numbers** can be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not zero. They include both **terminating decimals** and **repeating decimals**.



- **Irrational numbers** cannot be expressed in the form $\frac{a}{b}$.

Examples

FINDING SQUARE ROOTS OF PERFECT SQUARES

1. Find each square root.

A $\sqrt{49}$

$7^2 = 49$ *Think: 7 squared is 49.*

$\sqrt{49} = 7$ *Positive square root → positive 7*

B $-\sqrt{36}$

$6^2 = 36$ *Think: 6 squared is 36.*

$-\sqrt{36} = -6$ *Negative square root → negative 6.*

CLASSIFYING REAL NUMBERS

2. Write all classifications that apply to each real number.

A $\frac{8}{9}$

$8 \div 9 = 0.888... = 0.\overline{8}$

$\frac{8}{9}$ can be written as a repeating decimal.

rational number, repeating decimal

B 18

$18 = \frac{18}{1} = 18.0$

18 can be written as a fraction and a decimal.

rational number, terminating decimal, integer, whole number, natural number

C $\sqrt{20}$

$\sqrt{20} = 4.472135...$

The digits of $\sqrt{20}$ continue with no pattern.

irrational number

Order of Operations (for Holt Algebra 1, Lesson 1-6)

- The **order of operations** is a set of rules that tells what sequence to use when simplifying expressions that contain more than one operation.

Order of Operations	
First:	Perform operations inside grouping symbols.
Second:	Simplify powers.
Third:	Perform multiplication and division from left to right.
Fourth:	Perform addition and subtraction from left to right.

- Grouping symbols include parentheses ( ), brackets [ ], braces { }, fraction bars, radical symbols, and absolute-value symbols.
- If an expression contains more than one grouping symbol, simplify the innermost set first. Within each set, follow the order of operations.
- Grouping symbols may be used when translating from words to math. The product of 6 and the sum of 9 and 8 is written $6(9 + 8)$.

Examples

SIMPLIFYING NUMERICAL EXPRESSIONS

Simplify each expression.

1. $-4^2 + 24 \div 3 \bullet 2$

$-4^2 + 24 \div 3 \bullet 2$	*There are no grouping symbols.*
$-16 + 24 \div 3 \bullet 2$	*Simplify the power. The exponent belongs only to the 4.*
$-16 + 8 \bullet 2$	*Divide.*
$-16 + 16$	*Multiply.*
0	*Add.*

2. $\left|10 - 5^2\right| \div 5$

$\left\|10 - 5^2\right\| \div 5$	*The absolute-value symbols act as grouping symbols.*
$\left\|10 - 25\right\| \div 5$	*Simplify the power.*
$\left\|-15\right\| \div 5$	*Subtract within the absolute-value symbols.*
$15 \div 5$	*The absolute value of –15 is 15.*
3	*Divide.*

EVALUATING ALGEBRAIC EXPRESSIONS

3. Evaluate $21 - x + 2 \bullet 5$ for $x = 7$.

$21 - x + 2 \bullet 5$	
$21 - 7 + 2 \bullet 5$	*Substitute 7 for x. Then follow the order of operations.*
$21 - 7 + 10$	*Multiply.*
$14 + 10$	*Subtract.*
24	*Add.*

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