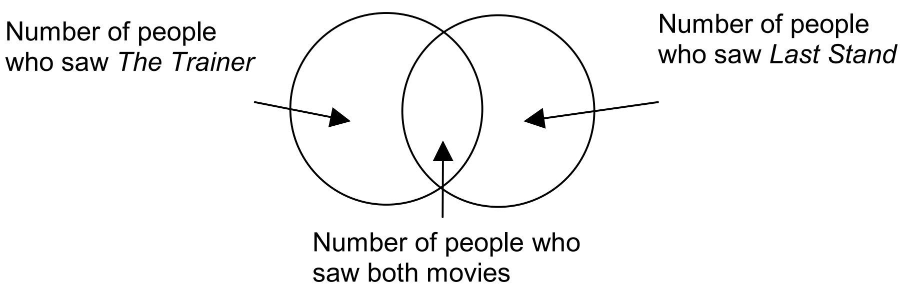
**PART II**

**Question 1: Key Idea 1A**

A total of 832 people went to the movie theater last night. Ticket sales show that 125 people saw *The Trainer*, 287 saw *Last Stand*, and 65 saw the double feature, which included both movies. Use a Venn diagram to determine the number of people who did not see either of these movies. Show how you arrived at your answer.

**Solution:**

Draw a Venn diagram, and let one circle represent the number of people who saw only *The Trainer* and another circle represent the number of people who saw only *Last Stand.* Overlap the two circles to represent the number of people who saw both movies.




**Determine how many people saw *The Trainer*.**

Ticket sales show that 125 people saw *The Trainer.* Of the 125 people, 65 saw both movies. To determine how many saw only *The Trainer*, calculate the difference between 125 and 65.

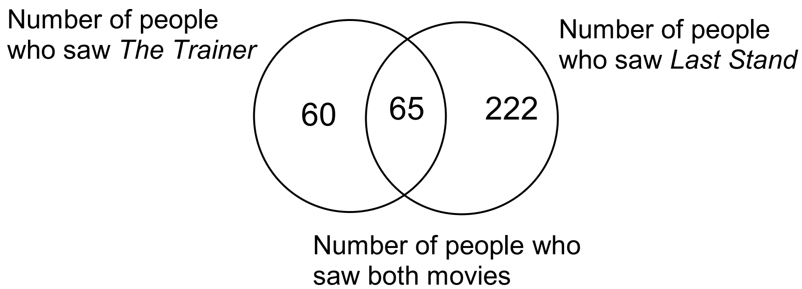
125 – 65 = 60
60 people saw only *The Trainer.*

**Determine how many people saw *Last Stand*.**

Ticket sales show that 287 people saw *Last Stand*. Of the 287 people, 65 saw both movies. To determine how many saw only *Last Stand*, calculate the difference between 287 and 65.

287 – 65 = 222
222 people saw only *Last Stand*.

**Fill in the Venn diagram with the appropriate numbers.**




**Find the total number of people who saw either movie.**

From the Venn diagram, you can see that 60 people saw *The Trainer*, 222 people saw *Last Stand*, and 65 people saw both movies. Add these numbers together to find the total number of people who saw at least one of the movies.

60 + 65 + 222 = 347
347 people saw at least one of the movies.

**Find the total number of people who did not see either movie.**

A total of 832 people went to the theater, and 347 people saw either *The Trainer* or *Last Stand*. Find the difference to determine how many people did not see either movie.

832 – 347 = 485
So 485 people did not see either movie.

**Question 2: Key Idea 2A**

1. Larry went to the store to buy fruit. He found that the store was selling six apples for $2.

If Larry wanted only to buy one apple, how much did it cost?

2. This week, Larry goes back to the store and finds that the apples are now on sale at a discounted price of six for $1. He decides to wait and come back in two weeks, and get them for free. Explain to Larry why he is mistaken, and tell him how much six apples will cost in two weeks and in four weeks.

**Solution:**

1. If 6 apples cost $2, set up a ratio to figure out how much 1 apple will cost.

$$\frac{\$2}{6} = \frac{\$x}{1}$$
$$6x = 2$$
$$x = \frac{2}{6} = \frac{1}{3} = 0.\overline{333}$$

Because you cannot pay $0.\overline{333}$ cents, round to the nearest cent. Therefore, one apple costs $0.34.

2. Larry believes that a possible pattern is that each week, six apples will cost one-half as much as they did the week before.

| | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 |
|---|---|---|---|---|---|
| **Price of Six Apples** | $2 | $1 | $0.50 | $0.25 | $0.125 |

| | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 |
|---|---|---|---|---|---|
| **Price of Six Apples** | $0.0625 | $0.0325 | $0.015625 | $0.0078125 | $0.00390625 |

Although the price of apples decreases by half and gets closer to zero, it will never be zero. Thus the apples will never really be free. Assuming that the price reduction of apples continues, if Larry waits two more weeks, six apples will cost him $0.25. If Larry waits four more weeks, six apples will cost him $0.07.

## Question 3: Key Idea 7A

28. Luke was asked to demonstrate how to solve the equation $3 + 2(x - 7) = 6x + 5$. Determine if each of Luke's steps is correct or incorrect, **based on the preceding equation**. If a step is incorrect, describe the error and explain what he should have done instead.

*Luke*

$$\begin{aligned} 3 + 2(x - 7) &= 6x + 5 \\ 5(x - 7) &= 6x + 5 \\ 5x - 35 &= 6x + 5 \\ -35 &= 11x + 5 \\ -30 &= 11x \\ -\frac{30}{11} &= x \\ -3\frac{8}{11} &= x \end{aligned}$$

**Solution:**

$3 + 2(x - 7) = 6x + 5$ Original equation.

$5(x - 7) = 6x + 5$ Incorrect; Luke added 3 + 2 first. He should have followed the order of operations and used the Distributive Property to multiply $2(x - 7)$. The step should be written as $3 + 2x - 14 = 6x + 5$.

$5x - 35 = 6x + 5$ Correct.

$-35 = 11x + 5$ Incorrect; Luke subtracted $5x$ on the left side and added $5x$ to the right side. He should have subtracted $5x$ from both sides. The step should be written as $-35 = x + 5$.

$-30 = 11x$ Incorrect; Luke added 5 to the left side instead and subtracted 5 from the right side. He should have subtracted 5 from both sides. The step should be written as $-40 = 11x$.

$-\frac{30}{11} = x$ Correct.

$-3\frac{8}{11} = x$ Incorrect; when changing the improper fraction to a mixed number, Luke divided incorrectly. $-\frac{30}{11} = -2\frac{8}{11}$.

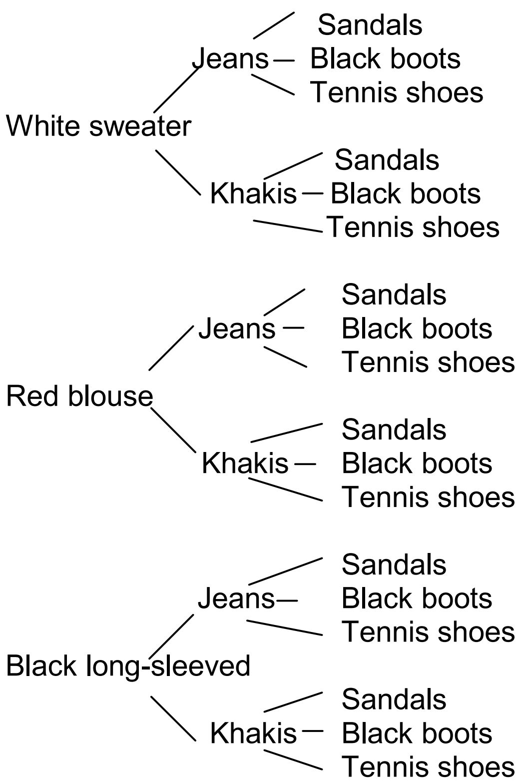
**Question 4:** **Key Idea 4A**

Maria plans to go with her friends to the movies but is not sure what she should wear. She has narrowed down her shirt options to a white sweater, a red blouse, or a black long-sleeved shirt. She figures she will wear either a pair of jeans or a pair of khaki pants. For shoes, she needs to decide between her new sandals, her black boots, and her favorite pair of tennis shoes.

Draw a tree diagram to illustrate all of Maria's possible outfits. How many possible outfits does she have? Determine the probability that she will wear the white sweater and khaki pants.

**Solution:**

**Tree Diagram**




White sweater, Jeans, Sandals
White sweater, Jeans, Black boots
White sweater, Jeans, Tennis shoes

White sweater, Khakis, Sandals
White sweater, Khakis, Black boots
White sweater, Khakis, Tennis shoes

Red blouse, Jeans, Sandals
Red blouse, Jeans, Black boots
Red blouse, Jeans, Tennis shoes

Red blouse, Khakis, Sandals
Red blouse, Khakis, Black boots
Red blouse, Khakis, Tennis shoes

Black long-sleeved, Jeans, Sandals
Black long-sleeved, Jeans, Black boots
Black long-sleeved, Jeans, Tennis shoes

Black long-sleeved, Khakis, Sandals
Black long-sleeved, Khakis, Black boots
Black long-sleeved, Khakis, Tennis shoes

Maria has a total of 18 different possible outfits.

**To find the probability that Maria will wear the white sweater and the khaki pants, use the probability formula.**

$$P(E) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{white sweater and khaki pants}) = \frac{3}{18} = \frac{1}{6} = 16.6\overline{6}\%$$

So there is a $\frac{1}{6}$, or $16.6\overline{6}\%$, probability that Maria will wear the white sweater and khaki pants to the movies.

**Question 5: Key Idea 5D**

Mrs. Benitez has 18 students in her Algebra class. Fifteen students took the final on Monday and received the following scores:
55, 77, 86, 85, 97, 40, 66, 72, 95, 70, 60, 82, 80, 75, and 70.

a. Define mean, median, mode, and range and create a table showing these values for the students' scores.

b. On Tuesday, the remaining three students took the final. They received scores of 65, 72, and 82. How do these scores change the mean, median, and mode of the test scores?

**Solution:**

**a.** Definitions:

The **mean** of the data set is the quotient when the sum of all of the elements is divided by the total number of elements.

The **median** of a data set is the middle number or the average of the two middle numbers in the set when the elements are placed in numerical order.

The **mode** of a data set is the element, if any, that occurs most often.

The **range** of a data set is the difference between the greatest and least values in the set.

**Find each measure of central tendency.**

List the numbers in increasing order.
40, 55, 60, 66, 70, 70, 72, 75, 77, 80, 82, 85, 86, 95, 97

**Mean:**

$$\frac{40+55+60+66+70+70+72+75+77+80+82+85+86+95+97}{15} = \frac{1110}{15} = 74$$

**Median:**
The number in the middle of the data set is 75.

**Mode:**
The score of 70 occurs the most often.

**Range:**
The greatest value is 97 and the smallest value is 40, so 97 – 40 = 57.

| Mean | Median | Mode | Range |
|---|---|---|---|
| 74 | 75 | 70 | 57 |

**b.** List the numbers in increasing order.

40, 55, 60, 65, 66, 70, 70, 72, 72, 75, 77, 80, 82, 82, 85, 86, 95, 97

**Mean:**
$$\frac{40+55+60+65+66+70+70+72+72+75+77+80+82+82+85+86+95+97}{18} = \frac{1329}{18} = 73.83$$

**Median:**
The number in the middle is the average of 72 and 75. $\frac{72 + 75}{2} = 73.5$

**Mode:**
The scores of 70, 72, and 82 all appear twice, which is the most often.

**Range:**
The greatest value is 97 and the smallest value is 40, so 97 – 40 = 57.

By adding Tuesday's test scores to the data set, the mean decreased by 0.17 to 73.83. The median decreased by 1.5 to 73.5. There are now three modes, 70, 72, and 82, and the range remained the same.

**PART III**

**Question 6: Key Idea 1B**

Consider the following conditional statement:

*If a parallelogram is a rectangle, then the parallelogram is a square.*

a. Determine whether the conditional statement is true or false. Explain your reasoning.
b. Write the converse of the conditional statement and determine if it is true or false. Explain your reasoning.
c. Write the inverse of the conditional statement and determine if it is true or false. Explain your reasoning.
d. Write the contrapositive of the conditional statement and determine if it is true or false. Explain your reasoning.

**Solution:**

**a.** The conditional statement is false because a rectangle is a quadrilateral with four right angles, and a square is a quadrilateral with four right angles *and* four congruent sides. If a parallelogram is a rectangle, it is not necessarily a square.

**b.** A conditional statement is in the form *if p then q*, or $p \Rightarrow q$. The converse of a statement is $q \Rightarrow p$. In other words, the conclusion of the original statement becomes the hypothesis of the converse, and the hypothesis of the original statement becomes the conclusion of the converse.

So the converse of the conditional statement is:

*If a parallelogram is a square, then the parallelogram is a rectangle.*

The converse of the conditional statement is true because a rectangle is a quadrilateral with four right angles, and a square is a quadrilateral with four right angles and four congruent sides. Therefore, a square is a rectangle.

**c.** The inverse of a statement is $\sim p \Rightarrow \sim q$. In other words, the negation of the original hypothesis is the hypothesis of the inverse, and the negation of the original conclusion is the conclusion of the inverse.

So the inverse of the conditional statement is:

*If a parallelogram is not a rectangle, then the parallelogram is not a square.*

The conditional statement is true because if a parallelogram is not a rectangle, then it does not have four right angles and therefore cannot be a square.

d. The contrapositive of a statement is $\sim q \Rightarrow \sim p$. In other words, the negation of the conclusion of the original statement becomes the hypothesis of the contrapositive, and the negation of the hypothesis of the original statement becomes the conclusion of the contrapositive.

So the contrapositive of the conditional statement is:

*If a parallelogram is not a square, then the parallelogram is not a rectangle.*

The conditional statement is false because a parallelogram can still be a rectangle even if it is not a square.

**Question 7: Key Idea 2B**

For what values of $x$ is each statement true? Provide examples in your explanations.

- $x^2 = x$
- $x^2 > x$
- $x^2 < x$

**Solution:**

**Determine the values of $x$ that make $x^2 = x$ true.**

There are only two values of $x$ that make $x^2 = x$ true: $x = 1$ and $x = 0$.

If $x = 1$, then $1^2 = 1 \bullet 1 = 1$.

If $x = 0$, then $0^2 = 0 \bullet 0 = 0$.

**Determine the values of $x$ that make $x^2 > x$ true.**

There are infinitely many values of $x$ that make this statement true.

For example,

if $x = 5$, then $5^2 = 5 \bullet 5 = 25$ and $25 > 5$;

if $x = –6$, then $(–6)^2 = (–6) \bullet (–6) = 36$ and $36 > –6$; and

if $x = -\frac{2}{3}$, then $\left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right) \bullet \left(-\frac{2}{3}\right) = \frac{4}{9}$ and $\frac{4}{9} > -\frac{2}{3}$.

Generally, any number greater than 1 and less than 0 makes $x^2 > x$ true.
So for $x < 0$ and $x > 1$, $x^2 > x$ is true.

**Determine the values of $x$ that make $x^2 < x$ true.**

There are infinitely many values of $x$ that make this statement true.

For example,

if $x = \frac{1}{2}$, then $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$ and $\frac{1}{4} < \frac{1}{2}$; and if $x = \frac{2}{3}$, then $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \bullet \frac{2}{3} = \frac{4}{9}$ and $\frac{4}{9} < \frac{2}{3}$.

Generally, any number less than 1 and greater than 0 makes $x^2 < x$ true.
So for $0 < x < 1$, $x^2 < x$ is true.

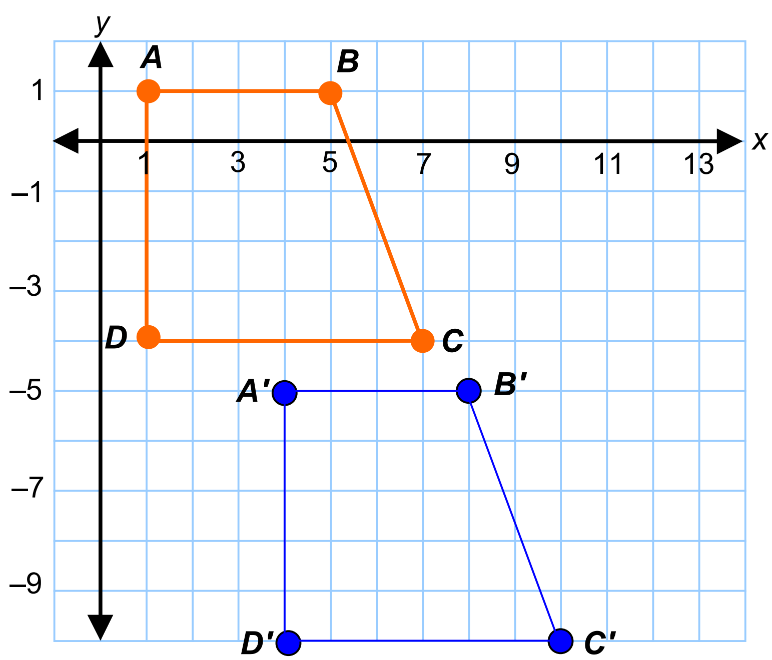
**Question 8:** **Key Idea 3C**

Trapezoid *ABCD* with coordinates *A*(1, 1), *B*(5, 1), *C*(7, –4), and *D*(1, –4) has been translated to trapezoid *A'B'C'D'* with coordinates *A'*(4, –5), *B'*(8, –5), *C'*(10, –10), and *D'*(4, –10).

- Graph both trapezoids on the same coordinate plane.
- Write and simplify expressions for the translation of the *x* and *y* coordinates of trapezoid *ABCD*.

**Solution:**

Graph trapezoid *ABCD* with coordinates *A*(1, 1), *B*(5, 1), *C*(7, –4), and *D*(1, –4). Graph trapezoid *A'B'C'D'* with coordinates *A'*(4, –5), *B'*(8, –5), *C'*(10, –10), and *D'*(4, –10).




**Determine the expression for the translation of the *x*-coordinates.**

| <u>Trapezoid *ABCD*</u> | ⇒ | <u>Trapezoid *A'B'C'D'*</u> | |
|---|---|---|---|
| 1 | ⇒ | 4 | 4 – 1 = **3** |
| 5 | ⇒ | 8 | 8 – 5 = **3** |
| 7 | ⇒ | 10 | 10 – 7 = **3** |
| 1 | ⇒ | 4 | 4 – 1 = **3** |
| ***x*** | ⇒ | ***x* + 3** | |

**Determine the expression for the translation of the *y*-coordinates.**

| Trapezoid *ABCD* | ⇒ | Trapezoid *A'B'C'D'* | |
|---|---|---|---|
| 1 | ⇒ | –5 | –5 – 1 = **–6** |
| 1 | ⇒ | –5 | –5 –1 = **–6** |
| –4 | ⇒ | –10 | –10 – (–4) = **–6** |
| –4 | ⇒ | –10 | –10 – (–4) = **–6** |
| ***y*** | ⇒ | ***y* – 6** | |

**Summary:**

For the translation from trapezoid *ABCD* to trapezoid *A'B'C'D',*
$(x, y) \Rightarrow (x + 3, y - 6)$.

**Question 9: Key Idea 4E**

The local high school choir is putting on a school musical. The choir is selling two types of tickets: adult and student. Adult tickets are \$6.25 each, and student tickets are \$4.50 each. There are 322 seats in the auditorium. The choir has already spent \$525 on props and paint, \$62 on advertising in the local paper, and \$88 on the choreographer. If all 322 seats are sold, how many of each type of ticket need to be sold in order for the choir to cover their expenses and make a profit of \$900?

**Solution:**

Let $x$ = number of adult tickets sold.
Let $y$ = number of student tickets sold.
Total tickets sold = 322 tickets
Price of adult ticket = \$6.25
Price of student ticket = \$4.50
Expenses = \$525 + \$62 + \$88 = \$675
Profit = \$900
Total money needed = \$675 + \$900 = \$1575

To solve the problem, set up a system of two equations with two unknowns.

**Equation #1:**

Number of adult tickets + Number of student tickets = Total tickets sold

$$x + y = 322$$

**Equation #2:**

(Adult ticket price)(Adult tickets sold) + (Student ticket price)(Student tickets sold) = Total cost

$$6.25x + 4.50y = 1575$$

**Solve the system.**

$x + y = 322 \rightarrow -4.50x + -4.50y = -1449$ Multiply all terms by –4.50.

$6.25x + 4.50y = 1575 \rightarrow \underline{6.25x + 4.50y = 1575}$

$1.75x + 0y = 126$

$1.75x + 0y = 126$ Solve for $x$.

$$\frac{1.75x}{1.75} = \frac{126}{1.75}$$

$$x = 72$$

$$\begin{aligned} x + y &= 322 \\ 72 + y &= 322 \\ -72 \quad & \quad -72 \\ y &= 250 \end{aligned}$$

Substitute the value of $x$ into one of the original equations.
Solve for $y$.

So 72 adult tickets and 250 student tickets need to be sold in order for the choir to make a profit of $900.

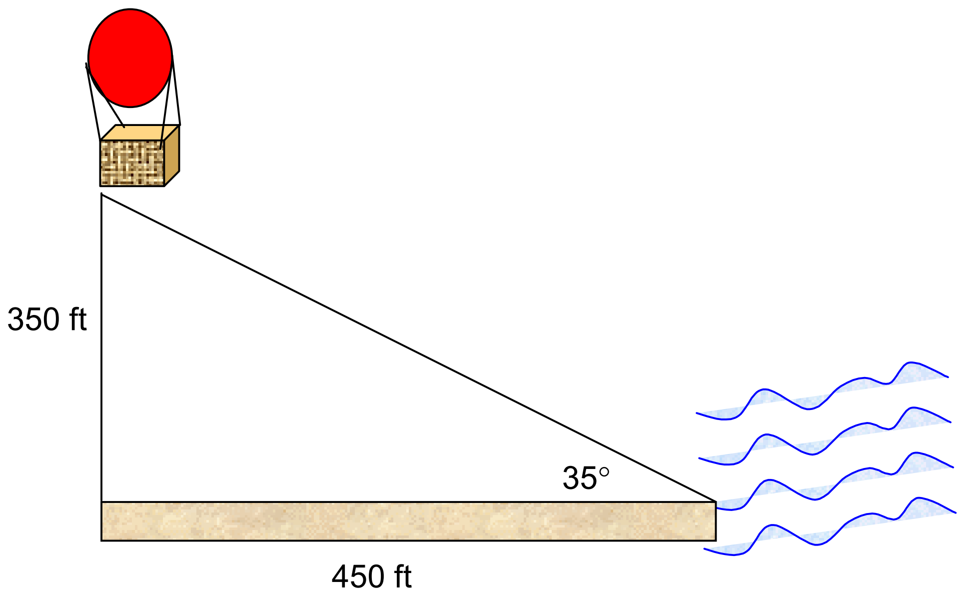
**Question 10:** Key Idea 5E

A hot air balloon is 350 feet above the ground and 450 feet from the water line at the beach. It is descending at a 35° angle relative to the ground. At this descent, will the hot air balloon land on the ground or in the water?

If the balloon is going to land on the ground, determine its distance from the water line. If the balloon is going to land in the water, find the greatest angle to the nearest whole number that the balloon could descend at instead and not get wet.

**Solution:**

**Draw a picture to help you understand the question.**




**Calculate where the balloon will land.**

Let $d$ = the distance to the landing point.

$$\tan 35 = \frac{350}{d}$$
$$d = \frac{350}{\tan 35}$$
$$d \approx 500$$

Since 500 feet is greater than 450 feet, the balloon will land in the water. So at the balloon's current descent of 35°, the balloon will get wet.

To avoid landing in the water, the balloon could come in at a steeper angle. To figure the greatest angle at which it could descend, solve the equation $\tan x = \frac{350}{450}$.

$$\tan x = \frac{350}{450}$$
$$\tan x \approx 0.7778$$
$$\tan^{-1} x \approx 0.7778$$
$$x \approx 37.87^\circ$$

A descent of approximately 38° relative to the ground will prevent the balloon from getting wet.

**PART IV**

**Question 11: Key Idea 6B**

How many of the integers from 1 through 400 are divisible by 2 or 3?
Find the probability that a random integer selected from 1 through 400 is divisible by neither 2 nor 3. Use the following table to help you to organize the data.

| **Numbers** | **Divisible by 2** | **Divisible by 3** |
|---|---|---|
| 1–10 | 2, 4, 6, 8, 10 | 3, 6, 9 |
| 11–20 | | |
| 21–30 | | |
| 31–40 | | |
| 41–50 | | |
| 51–60 | | |
| 61–70 | | |
| 71–80 | | |
| 81–90 | | |
| 91–100 | | |

**Solution:**

**Determine the number of integers from 1 to 100 that are divisible by 2 or 3 to determine a pattern.**

| **Numbers** | **Divisible by 2** | **Divisible by 3** |
|---|---|---|
| 1–10 | 2, 4, 6, 8, 10 | 3, 6, 9 |
| 11–20 | 12, 14, 16, 18, 20 | 12, 15, 18 |
| 21–30 | 22, 24, 26, 28, 30 | 21, 24, 27, 30 |
| 31–40 | 32, 34, 36, 38, 40 | 33, 36, 39 |
| 41–50 | 42, 44, 46, 48, 50 | 42, 45, 48 |
| 51–60 | 52, 54, 56, 58, 60 | 51, 54, 57, 60 |
| 61–70 | 62, 64, 66, 68, 70 | 63, 66, 69 |
| 71–80 | 72, 74, 76, 78, 80 | 72, 75, 78 |
| 81–90 | 82, 84, 86, 88, 90 | 81, 84, 87, 90 |
| 91–100 | 92, 94, 96, 98, 100 | 93, 96, 99 |

Each group of ten has five integers divisible by 2 and either three or four integers divisible by 3. Every third group has four integers divisible by 3.

In order to not count a number twice, if an integer divisible by 2 is also divisible by 3, delete that integer from the table.

| Numbers | Divisible by 2 | Divisible by 3 |
|---|---|---|
| 1-10 | 2, 4, 6, 8, 10 | 3, 6, 9 |
| 11-20 | 12, 14, 16, 18, 20 | 12, 15, 18 |
| 21-30 | 22, 24, 26, 28, 30 | 21, 24, 27, 30 |
| 31-40 | 32, 34, 36, 38, 40 | 33, 36, 39 |
| 41-50 | 42, 44, 46, 48, 50 | 42, 45, 48 |
| 51-60 | 52, 54, 56, 58, 60 | 51, 54, 57, 60 |
| 61-70 | 62, 64, 66, 68, 70 | 63, 66, 69 |
| 71-80 | 72, 74, 76, 78, 80 | 72, 75, 78 |
| 81-90 | 82, 84, 86, 88, 90 | 81, 84, 87, 90 |
| 91-100 | 92, 94, 96, 98, 100 | 93, 96, 99 |

From the integers 1–100, there are 34 integers divisible by 2, 17 integers divisible by 3, and 16 integers divisible by both 2 and 3. There are 67 integers divisible by 2 or 3.

**Use the pattern to determine how many integers from 1 through 400 are divisible by 2 or 3.**

If there are 67 integers from 1 through 100 divisible by 2 or 3, this is the same for each group of 100. So there are 4(67) = 268 integers from 1 through 400 divisible by 2 or 3.

**Determine the probability that a random integer selected from 1 through 400 is divisible by neither 2 nor 3.**

If there are 268 integers from 1 through 400 that are divisible by 2 or 3, then there are 400 – 268 = 132 integers that are divisible by neither 2 nor 3.

*P*(integers neither divisible by 2 or 3) = $\frac{132}{400} = \frac{33}{100} = 0.33.$

So there is a 33% probability that a random integer selected from 1 through 400 is divisible by neither 2 nor 3.

**Question 12: Key Idea 6D**

There are 10 nominees for the high school prom committee.

**a.** How many ways are there to choose a committee of 5 people from a group of 10 people?

**b.** How many ways are there to choose 3 separate officeholders (chairperson, secretary, and treasurer) from a group of 10 people?

**Solution:**

**a.** The members chosen for the committee will be members regardless of the order in which they are chosen.

The number of combinations of *n* objects taken *r* at a time is given by the following formula:

$$C(n,r)=\binom{n}{r}=\frac{n!}{r!(n-r)!}, 0 \le r \le n.$$

Find $C(10, 5)$.

$$C(10,5)=\binom{10}{5}=\frac{10!}{5!(10-5)!}=\frac{10!}{5!5!}=\frac{10\bullet 9\bullet 8\bullet 7\bullet 6}{5\bullet 4\bullet 3\bullet 2\bullet 1}=\frac{30,240}{120}=252$$

There are 252 ways to choose a committee of 5 people from a group of 10 people.

**b.** The officeholders are chosen to fulfill particular positions. Therefore, order is important.

Find $P(10, 3)$.

$$P(10,3)=\frac{10!}{7!}=10\times 9\times 8=720$$

There are 720 ways to choose a chairperson, a secretary, and a treasurer from a group of 10 people.

**Question 13: Key Idea 7C**

New agents at H-1 Realty are allowed to choose between two different payment plans. Plan 1 offers a base salary of $3000 per month and a 4% sales commission. Plan 2 offers no base salary but an 8% sales commission.

1. Write an equation for the total monthly pay for each plan.
2. Graph each equation on the same coordinate plane.
3. Determine which plan a new agent should choose if he plans to sell $120,000 in property per month.

**Solution:**

**1. Write an equation.**

Let $x$ represent the monthly sales of a new agent at H-1 Realty.

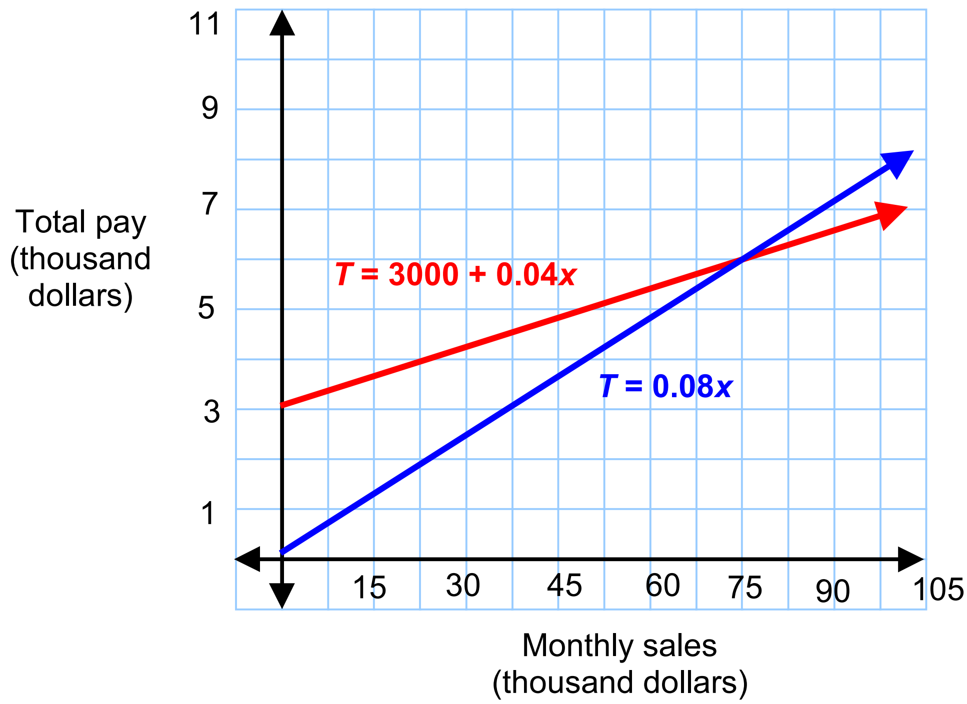
**Plan 1**
Total pay = base pay + commission
Commission = (percent)(monthly sales)

Total pay = base pay + (percent)(monthly sales)
Total pay = 3000 + (0.04)(monthly sales)
$\boldsymbol{T = 3000 + 0.04x}$

**Plan 2**
Total pay = commission
Commission = (percent)(monthly sales)

Total pay = (percent)(monthly sales)
Total pay = (0.08)(monthly sales)
$\boldsymbol{T = 0.08x}$

**2. Graph each equation.**




**3. Determine which plan a new employee should choose.**

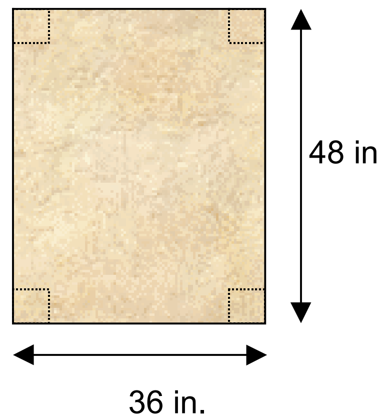
By looking at the graph of the two equations, you can see that if the new agent sells more than $75,000 per month, plan 2 will result in more money. So if a new agent plans to sell $120,000 in property per month, then plan 2 should be chosen.

**Question 14:** **Key Idea 7E**

Jerry is rebuilding the engine of his car and is spilling grease onto his parents' driveway. He needs to think of something quick to prevent more grease from spilling over onto the concrete. He gets a piece of cardboard now, but it is does not work well. Something with edges is needed to contain the grease. He has another piece of cardboard that is 36 inches by 48 inches. If he cuts out squares from each corner and folds up the sides, he can contain the grease.

Write an expression for the area of concrete that the cardboard will cover after the corners are folded up.

If Jerry needs the cardboard to cover 448 in² of concrete, what dimensions should the squares be? Explain why only one out of the two solutions is reasonable.

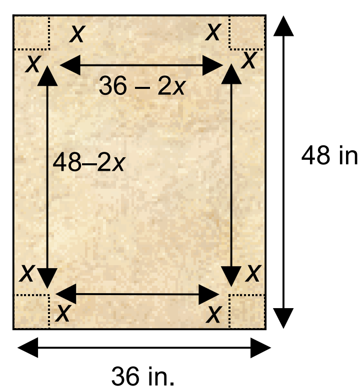



**Solution:**

**Write an expression for the area.**

The area of a rectangle is the product of its length and width. Since the dimensions of the squares are unknown, let the side of each square be $x$.

Once these squares are cut out, the area of the rectangle changes. The length becomes $48 – 2x$ instead of 48. The width becomes $36 – 2x$ instead of 36.




The area is now the product of the new length and the new width.

$A = (48 – 2x)(36 – 2x)$

**Determine *x* if the given area needs to be 448 in².**

Given $A = 448$ and $A = (48 - 2x)(36 - 2x)$, set the equations equal to each other and solve for $x$.

| | |
|---|---|
| $448 = (48 - 2x)(36 - 2x)$ | Solve the equation for *x*. |
| $448 = 1728 - 96x - 72x + 4x^2$ | Multiply the binomials. |
| $448 = 1728 - 168x + 4x^2$ | Add like terms. |
| $0 = 1280 - 168x + 4x^2$ | Subtract 448 from both sides. |
| $0 = 4x^2 - 168x + 1280$ | Use the Commutative Property to rearrange the quadratic expression. |
| $0 = 4(x^2 - 42x + 320)$ | Factor out 4 from the quadratic expression. |
| $0 = x^2 - 42x + 320$ | Divide both sides by 4. |
| $0 = (x - 10)(x - 32)$ | Factor the quadratic expression. |
| $x - 10 = 0$ or $x - 32 = 0$ | Use the Property of Zero to write two equations. |
| $x = 10$ or $x = 32$ | Use the Addition Property of Equality to solve both equations. |

The dimensions of the squares need to be 10 inches by 10 inches. It is unreasonable for the dimensions of the squares to be 32 inches by 32 inches because the width of the original piece of cardboard is only 36 inches and therefore not long enough for Jerry to cut out two 32-inch by 32-inch squares.

**Question 15: Key Idea 3B**

Given the expression $a^n$, where *a* is any integer and *n* is any positive integer, write a rule that can be used to determine the signs of the expression. Remember to include all cases.

**Solution:**

**List facts.**

- If *a* is any integer, then *a* is a positive or negative integer or zero.
- If *n* is any positive integer, then *n* is not equal to zero.
- The product of an even number of negative numbers is positive.
- The product of an odd number of negative numbers is negative.
- The product of any number of positive numbers is positive.

**Determine the number of cases.**
Four cases need to be addressed:

1. when *a* is positive
2. when *a* is negative, and *n* is an even number
3. when *a* is negative, and *n* is an odd number
4. when *a* is zero

**Write the rule.**
Case 1: $a^n$ is positive because the product of any number of positive numbers is positive. Let the symbol + represent a positive number.

$$a^n = (+)^+ = (+)(+)(+)(+)(+)(+)\ldots(+) = +$$

Case 2: $a^n$ is positive because the product of an even number of negative numbers is positive. Let the symbol + represent a positive number and the symbol – represent a negative number.

$$a^n = (-)^{\text{even}} = \underbrace{(-)(-)(-)(-)(-)\ldots(-)}_{\text{even number of times}} = +$$

Case 3: $a^n$ is negative because the product of an odd number of negative numbers is negative. Let the symbol + represent a positive number and the symbol – represent a negative number.

$$a^n = (-)^{\text{odd}} = \underbrace{(-)(-)(-)(-)(-)(-)\ldots(-)} = -$$

Case 4: When *a* is zero, $a^n$ is zero since zero to any power is zero.